Lorentz Covariance as a Dynamic Symmetry

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October, 2012

Let a local observer interact with the world and map physical events in an inner space-time, the observer's frame of reference. This inner space-time, being located within a single point of the outer space (the current position of the observer), is essentially non-local, so that different locations can be compared within the same reference frame. In the way, for instance, the observer can detect any outer motion, which becomes represented by a sequence of transitions in the inner space-time; this allows the observer to measure velocities of the outer objects relative to the current frame of reference. In particular, one can determine the relative velocities of different observers. However, in this context, we do not care for the possible comparison of the physical pictures obtained in different reference frames. All we need is a single observer capable of building some physics on the basis of local observations.

Basically, each physical process is described by some quantity, and the physical sense of such numeric representations is in a number of qualitative effects revealing the global features of the process. Such overall properties of physical systems are known as symmetries. Any symmetry means that we can transform both the coordinate system and the physical quantities without changing the global parameters characterizing the symmetry. The number and possible values of such invariants are determined by the nature of the object.

Thus, many physically important interactions are invariant in respect to spatial or temporal translations (homogeneity of space and time). If this symmetry becomes violated, we can either suspect a non-inertial motion of observer, or admit the presence of a physical interaction essentially distinguishing the spatial points by assigning them the different values of field potential, and possibly non-stationary in time. Similarly, isotropy leaves some interactions invariant in respect to spatial rotations. There are also discrete symmetries both space-related (like parity or time reversal) and referring to inner motion (like spin and isospin).

The presence of a particular symmetry is not a question of abstract speculation; this is what we learn in our practical activity in the real world. The organization of the inner space of each observer is determined by this practical experience. However, since any individual activity is finite, there is no guarantee that a symmetry observed in a limited area during a limited time will necessarily hold on a wider scale; conversely, certain global symmetries can merely be an averaged picture of a less symmetric lower-level motion. It is important that the inner representation of a physical system in a given frame of reference depends both on the nature of the system and the structure of the reference frame. The latter may differ from any physical structures, thus leading to various "spurious" effects, or "artefacts". This does not necessarily mean the inadequacy of the corresponding activity and a need of change. A deliberate choice of a non-inertial reference frame may be useful to simplify theoretical models and get to the final result in a more transparent manner. Of course, we must be aware of the possibly "unphysical" effects and be careful in our conclusions, never exaggerating their universality. Anyway, there is no absolutely preferable frame of reference, since any frame can only remain inertial within a specific physical context, on a definite level of hierarchy. That is, all our theories will necessarily bear a mark of artificiality, and we should not take them for a final and absolute truth. The world is much wider than any human experience, and the development of the human culture is bound to go beyond the bounds of any science, including physics.

Now, let us consider the organization of a model space-time, which, in a given inertial frame of reference, is represented by the real numbers x and t, the spatial and temporal coordinates. For simplicity, only one spatial dimension has been left. The exact values of coordinates are irrelevant to any physics, merely characterizing the specific way of introducing a formal structure in the inner space of the observer. Obviously, the observer can choose any other coordinates, while the overall physical picture should not depend on such inner transformations. As we do not yet have any other quantities, the possible invariants must be constructed from the coordinates x and t.

The simplest way to combine these quantities is a power expansion. However, in the assumption of homogeneity, the presumed symmetry parameter will only depend on the distances dx and durations dt, and, in the assumption of isotropy, only the even powers of these displacements are to be included. The latter assumption is rather strong and non-trivial. Thus, we could explicitly account for the irreversibility of time, leaving isotropy for spatial rotations and mirror reflection. This is how non-relativistic quantum mechanics is organized. On the other hand, the presence of the second order terms like $dx\,dt$ (or the zero-order terms like dx/dt) demands the synchronicity of mirror reflections changing the sign of both spatial and temporal coordinates. Any independent mirror symmetry will result in a violation of the overall invariance. To avoid such issues, let us define the *interval* in the current frame of reference as

$$ds^2 = c^2 dt^2 - dx^2 ,$$

where c is some velocity needed to bring the two terms to the same dimension, thus allowing their direct addition. Obviously, in a homogeneous and isotropic space, this velocity does not depend on any coordinates. The minus sign has been chosen just to stress the difference of spatial and temporal dimensions; otherwise, we would obtain an entirely static Euclidean plane.

To make this formal construct meaningful, we need to demand its invariance under some general transformations of the coordinate system *within the same reference frame* (that is, this symmetry has nothing to with the relative motion of different observers). So, let the interval be invariant in respect to some family of linear transforms:

$$\tilde{c}\,d\tilde{t} = Acdt + Bdx$$
$$d\tilde{x} = Tcdt + Xdx$$

Once again, we have introduced dimensional factors to simplify calculations. Also, for generality, we admit that the dimensional factor c is not necessarily invariant on itself, changing, if necessary, to ensure the overall invariance.

Note that any transform at all is necessarily *global*, it refers to *the whole* inner space of the observer (reference frame). Considering any inaccessible regions, or singularities, would contradict the very existence of a frame of reference as a coherent construct to represent outer motion. Also, the considerations of homogeneity lead us to using distances rather than coordinates, while the coefficients of the transform remain constant all over the reference frame. Of course, nothing prevents us from treating them as the functions of a specific location; however, physically, this is equivalent to splitting the frame of reference into a family of local frames, that is, considering many independent observers, which effectively brings us to a different level of hierarchy.

Substituting the above equations into the invariance condition

$$\tilde{c}^2 d\tilde{t}^2 - d\tilde{x}^2 = c^2 dt^2 - dx^2 ,$$

we obtain the relations between the coefficients of the admissible transforms:

$$A^2 = B^2 + 1$$
$$X^2 = B^2 + 1$$
$$T^2 = B^2$$

Consequently, a linear interval-conserving transform must be of the following general form:

$$\begin{aligned} \tilde{c}d\tilde{t} &= \sqrt{B^2+1}\cdot cdt + Bdx \\ d\tilde{x} &= Bcdt + \sqrt{B^2+1}\cdot dx \end{aligned} \tag{BL}$$

This is a one-parametric family, and the parameter B may take any numeric values, either real or complex. The sign of the transform parameter A has been chosen to ensure that $\tilde{c}d\tilde{t} = cdt$ and $d\tilde{x} = dx$ for $B = 0$.

It is important that the parameter B in (BL) is entirely unrelated to the dimensional factor c, and hence the transformation law for the scaling factor is irrelevant for the conservation of interval and must be determined from other considerations. In principle, nothing prevents us from also considering complex valued dimensional factors (though we won't dwell on it any more). In the most general case, $\tilde{c} = f(c, B)$, with a smooth enough arbitrary function f. For instance, one could choose $\tilde{c} = c$, thus making c a physical parameter, yet another invariant of the same symmetry. This parameter is readily interpreted as a kind of the space-time binding strength in a given frame of reference (though it may change from one reference frame to another).

Now, let us make a (singular) parameter substitution

$$B = \frac{\beta}{\sqrt{1-\beta^2}}$$

$$\sqrt{B^2+1} = \frac{1}{\sqrt{1-\beta^2}}$$

The inverse function is defined as

$$\beta = \frac{B}{\sqrt{1+B^2}},$$

so that, obviously, $\beta \neq 1$ for any finite B. The re-parameterized transform is

$$\begin{aligned} \tilde{c}d\tilde{t} &= \frac{cdt + \beta dx}{\sqrt{1-\beta^2}} \\ d\tilde{x} &= \frac{dx + \beta cdt}{\sqrt{1-\beta^2}} \end{aligned} \tag{βL}$$

which closely resembles the well-known Lorentz transform, the basis of relativistic physics. That is why we can refer to the invariance of the interval under the transform (BL), or (βL), as the Lorentz symmetry. However, there is a significant difference. All our derivation refers to a dynamic symmetry existing *in the same frame of reference*, and we did not yet consider any issues of relative motion. In this respect, this symmetry is like any other *physical* symmetries (homogeneity, isotropy, mirror symmetry), which refer to the character of dynamics and hence, according to the principle of relativity, independently hold for each observer. Of course, we can formally set

$$\beta = \frac{v}{\bar{c}},$$

with a new global constant $\bar{c}$, which does need to correspond to the already available dimensional factor c; anyway, the velocity v is nothing but a transform parameter unrelated to any outer motion. If, for some reasons, we set $\tilde{c} = c$, then it would be quite natural to also set $\bar{c} = c$, which is equivalent to merely rescaling the parameter v.

The transforms (BL) and (βL) are defined for any values of the parameters B and β, respectively.

In particular, the form (βL) does not necessarily imply the traditional "relativistic" restriction $\beta < 1$. Thus, for $\beta > 1$, we can define a new parameter

$$\alpha = \frac{1}{\beta} = \frac{\bar{c}}{v}$$

and obtain

$$B = -\frac{i}{\sqrt{1-\alpha^2}}$$
$$\sqrt{B^2+1} = \frac{i\alpha}{\sqrt{1-\alpha^2}}$$

so that

$$icd\tilde{t} = \frac{dx - \alpha c dt}{\sqrt{1-\alpha^2}}$$
$$id\tilde{x} = \frac{cdt - \alpha dx}{\sqrt{1-\alpha^2}} \qquad (\alpha L)$$

These equations may produce an impression of the former spatial coordinate becoming a new time variable, while the former time becomes the new spatial coordinate in the region $v > \bar{c}$. In a relativistic world, this might provoke a range of speculations on the peculiarities of the tachyon world. But in our approach, the singular forms (βL) and (αL) have both been obtained from the regular form (BL), which does not contain any singularities. The limit $\beta = 1$ correspond to $B \to \infty$, while the limit $\alpha = 1$ correspond to $B \to -i\infty$, so that the real and imaginary branches are entirely unrelated to each other.

The appearance of spurious singularities is a natural property of any singular transformation. Since the interval (–∞, ∞) is isomorphic to the interval (–1, 1), we are free to map one infinity onto another. In this case, it would make no sense asking what the world would be like outside the interval (–1, 1); this is exactly equivalent to the question about what happens if we go a little bit farther than infinity. For comparison, regardless to any motion at all, we can pack our Euclidean space into a unit cube, and this singular transformation will raise an impenetrable boundary for all those inside the cube to wonder what the world is like there, behind the wall. Similarly, spurious singularities can appear in various nonlinear transformations—for instance, introducing the ordinary polar coordinates. But this would lead us to discussing the logic of general relativity, which is beyond the scope of this note.

If two observers S and S' move relative each other at a constant speed V, the interval can be defined in either frame of reference and be a local symmetry (like homogeneity or isotropy) regardless of the other observer. The formal parameter B of the symmetry transform (BL), or β in (βL), is entirely local, and there is no need to consider any correspondence of such arbitrary parameters between different frames.

The absolute values of both the interval ds^2 and the coupling parameter c depend on the choice of the units of length and duration. In general, there is no reason why these units could not change from one reference frame to another, unless there is a specific way of establishing a regular correspondence, suggesting a real physical procedure to construct comparable frames. Indeed, even if we decide that all the observers should measure length in meters, and agree on the identic procedure of measurement to map the outer motion into the inner space, there is no obvious way to tell whether the meter in one frame of reference will be the same as in another frame, moving relative to the first. This is much the same as trying to determine the exact value of money: for instance, we may have both euros and US dollars, assuming a definite exchange rate; however, there are other people who use the Russian ruble, or the Chinese yuan, and they will find that that our exchange rate is different from the cross rate they obtain separately converting euros and dollars into their native currency. Adding more currencies (or measuring the value of money in the quantities of certain goods) makes the problem even more complicated, since neither measure is preferable.

One possible solution is to *postulate* the equal values of c and ds^2 in any frame of reference. This will fix the units of measurement up to an overall scaling factor (that is, multiplying the units of length and duration by the same number in all the frames of reference). Note that such a postulate does not reflect any physics at all; it is a mere convention about the units employed. Physical considerations are introduced here in an implicit manner, through the choice of the type of symmetry serving as the common reference. In the above economic analogy, this would mean an arbitrary choice of two preferred currencies, with a fixed "standard" exchange rate; in this case, the "absolute" value of all the other currencies can be expressed through any of the basic two. This is a non-trivial economic fact, since it allows deriving the entire economic theory from the balance of the two "fundamental" values (just as Karl Marx did in his "Das Kapital"). Similarly, all the physical relativism is basically due to a choice of the preferred units of measurements. That is, if we choose to compare everything with light, the constancy of the speed of light will be a *logical* demand. The important corollary is that we are to obtain a different picture of the world with any other choice of the reference process, provided it is related to light in a nontrivial way. But this will be discussed elsewhere.

So far, let us demand the constancy of the coupling constant c and the invariance of the interval ds^2 in all the inertial frames of reference. Assuming that the coordinates in a moving frame are related to the static coordinates by a linear transform (which, in general, is not an obvious solution), we can apply the same logic as with the local interval conservation and obtain a similar transform:

$$\begin{aligned} c d\tilde{t} &= \sqrt{q^2+1}\cdot cdt + qdx \\ d\tilde{x} &= qcdt + \sqrt{q^2+1}\cdot dx \end{aligned} \tag{qL}$$

The parameter $q = q(c, V)$ is an *arbitrary* function of the speed of the relative motion V, being entirely unrelated to the parameter B in the inner symmetry transform (BL). In other words, constancy of the coupling speed and the preservation of the interval in different reference frames does not imply a specific coordinate transform, but rather a family of admissible transforms. Thus, one could make a trivial choice: $q = 0$ (which is a legal function of V, isn't it?). In this case, we readily get

$$\begin{aligned} d\tilde{t} &= dt \\ d\tilde{x} &= dx \end{aligned} \tag{GL}$$

exactly reproducing the classical Galilean picture; the value of the interval will obviously be the same in all the reference frames, since nothing really changes. Note that it is does not influence the fundamental symmetry, the invariance under the inner transform (BL). It seems like the constancy of the speed of light does not at all contradict the traditional Newtonian physics, as we have been used to believe since the first Einstein's papers on relativity. Or, rather, their interrelation is a little bit more complex than expected.

Of course, the traditional Lorentz transform is yet another possible choice:

$$\begin{aligned} c d\tilde{t} &= \frac{cdt + \beta(V)dx}{\sqrt{1-\beta(V)^2}} \\ d\tilde{x} &= \frac{dx + \beta(V)cdt}{\sqrt{1-\beta(V)^2}} \end{aligned} \tag{L}$$

In this form, the transform allows generalizations, assuming that $\beta(V)$ varies from 0 to 1, as V varies from 0 to some upper limit V_{max}, which may be set to c, but may also take any other value, and even pushed to infinity.

This essential arbitrariness may seem astonishing, and probably upsetting, in the common view of physics as a precise science that must give unambiguous answers to practical problems. On the same reasons, some physicists do not like the infinity of landscapes suggested by the string theory; on the same reasons, Einstein treated quantum mechanics as insufficient. One could expect that the choices in (qL) would be significantly narrowed by additional physical considerations, effectively imposing more restrictions on the system's dynamics. Still, some versatility may remain, and this could be a physical consideration as well, admitting that nature is hierarchically organized, and the observable picture is different on the different levels of this hierarchy. Some of them will correspond to classical or relativistic mechanics, while the others may open new promising directions of scientific and technological development.

As one could already have guessed, the symmetry imposed by interval preservation is related to the symmetry of classical electrodynamics. That is, we take electromagnetic processes as a common measure for the spatial and temporal aspects of any other physics. This may well be justified by the special role light plays in the real world; but even in this case, there is some freedom in theoretical description that might be used to achieve more integrity in the physical science. In a way, the situation is analogous to the history of the famous fifth postulate in Euclidean geometry that could be removed without any damage to the theory's consistency. Einstein's approach is, probably, not the only possible kind of relativity. Why not indulge our curiosity and play with the other alternatives as well?

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