Hierarchy of Logic

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> ***Warning:*** the language of this work is different from that of any mathematical theories (and category theory in particular), though some constructions may remind them. No identifications should be made with the other philosophy texts. The existing parallels are to be considered separately.

Hierarchy of creativity

Three basic levels: art, science, philosophy.

The principal task of ***art*** is to produce forms. This is the first abstraction of the form from the activity, which is necessary to combine forms in an arbitrary manner, and thus obtain yet unknown combinations. However, art does not invent forms, it extracts them from the currently known activities.

Combining abstract forms in order to generate new forms that are not directly related to the known activities is performed by ***science***. The resulting combinations may be used in further transformations, producing higher-order abstractions.

Philosophy serves to link the abstractions of science back to activity. It indicates how the achievements of science are to be transformed into practice.

Creativity may be a component of any activity. In certain cultural circumstances, there may be special activities representing art, science and philosophy. However, creativity can never become an activity, and such a specialization is to be removed on a higher level of development.

Reflectivity

Since art, science and philosophy are realized in specific activities, they may become the starting point for extracting the respective forms in art, constructing new scientific abstractions, and developing philosophic principles applicable in practice.

Thus the levels of creativity become reflected in each other. In particular, philosophy influences both art and science, as well as its own development. However, this influence is not direct, and philosophy as a regulator of artistic, scientific or philosophical creativity appears in the transmuted forms. On the level of art it becomes ***aesthetics***. In science, it manifests itself as ***logic***. As a determiner of the development of philosophy it may be called ***ethics***.

In the same way, art becomes represented on the levels of science and philosophy, and science penetrates philosophy and art.

Levels of logic

Because of reflectivity, there are different levels of science:

1. Knowledge may be merged with activity, contained in its historically formed schemes, apprehended through learning. This is the syncretic level of science.
2. With the development of society, science may become associated with a separate activity appropriated by a special social group, the professional scientists. This is what is commonly meant under the word “science”, its analytic level.

3. However, science has to finally become a part of practice, and thus return to the people's everyday life. This level may be called synthetic, and one might suggest engineering as a representative.

Accordingly, there are different levels of logic:

1. Syncretic level: ***insight***. The formation of the conceptual basis for the higher levels of logic Intuitively accepted notions and rules, the choice of the direction of activity.
2. Analytic level: ***reasoning***. Formal inferences. Explicit rules. Intellectual operations.
3. Synthetic level: ***understanding***. Both intuition and reasoning combined for a practical purpose. Ingenuity.

Traditionally, the scientific study of logic was restricted to the analytic level. However, the major part of every science (including mathematics) is not explicitly formalized, and it the consideration of the syncretic and synthetic levels of logic is required as well.

Logic and reflectivity

Since logic does not belong to the level of science, it cannot be completely comprehended through scientific research. Logic develops with the development of culture, which virtually reflects the development of the world. Hence, there are no absolute logical rules, valid for all the cultures and all the times. In relation to development, the following kinds of logic may be distinguished:

1. ***Rationality***. Stationary activities, with stable schemes existing for a long time. At the syncretic level, this is common sense. Analytical rationality includes Aristotelian syllogistics and the logical systems described in mathematics. In philosophy, this way of thinking is called metaphysical.
2. ***Dialectics***. Every action is viewed in a broader context, along with its alternatives. Struggle and mutual reflection of the opposites, their unity being achieved on a higher level. Syncretic dialectics: pragmatism. Analytical dialectics: sophism. Synthetic dialectics was developed in Marxism.
3. ***Diathetics*** ("intentional arrangement"). Deep reflectivity of any category, so that every one of them contains all the other categories in it and the whole may be reconstructed starting from anywhere. The infinite sequences of levels assumed by dialectics (the higher ones fixing the contradictions of the lower) are just different manifestations of the whole, the unfoldings of a hierarchy. Hegel's "speculative logic" is an example of a weak form of diathetics restricted to the systematic study of philosophic categories only.

Logical coherence

There are three levels of coherence: structure, system, hierarchy.

1. ***Structure***: elements and links, simultaneity.
2. ***System***: one structure (input) transformed into another (output); dynamics.
3. ***Hierarchy***: systems as the elements of a structure, structures as the implementations of a system; development.

One may consider logical coherence, distinguishing logical structures (states), logical systems (processes) and logical hierarchies (development). On the other side, each level of coherence assumes its own ways of operation and reasoning, and there are structural logic, systemic logic and hierarchical logic.

Every particular activity (or reasoning) combines all the three levels of logical coherence. However, different cultures accentuate different kinds of logic, and there may be practical tasks requiring the domination of the structural, systemic or hierarchical view.

Logical universality

In general, every kind of activity obeys its own logic. However, in every culture, some schemes are of a wider applicability than the others, and there are hierarchical relations between logical schemes. One scheme may be obtained from another — and a simple scheme may become the origin of numerous schemes, up to very complicated.

However, there is no absolute ordering of logical schemes by the level of complexity. In principle, any activity may be made a pattern for many other activities, and the same logical scheme may be obtained in many ways. The unity of logic may be inferred from the unity of the world, and hence every two logical schemes can be transformed into each other using the appropriate logical means. The variability of axiomatic systems in mathematics is a typical example.

Any logical scheme is universal, and the scope of its applicability mainly depends on the cultural factors. Since logic refers to the analytical level of creativity, there are no restrictions on formal manipulations, and one inference cannot be more logical than another — they may only correspond to the different logical schemes. However, some schemes are more preferable in the practice than the others, which is virtually related to the level of cultural development.

Because of reflectivity, logic contains any knowledge at all. In particular, all the science may be considered as an unfolded form of logic. However, this relation can be inverted, so that any activity (and any science) will contain all the logical schemes. Logic becomes implemented in the practice, while the practice gives birth to logic.

Logic and culture

The development of the world is marked by a definite directedness. The human society is one of the necessary levels of this development, and the forms of subjectivity reflect the hierarchy of the world. In particular, the logical forms of any society or social group correspond to its place in the formation of a larger scale. There are no "truths" equally acceptable by everybody in any situation. Any general norms are due to the similarity of the positions in some common activity.

Every objectively formed social group is characterized by a common logic, reflecting the place of that group in the society. This implies the communication of the logical schemes from one person to another, learning logic. However, such learning cannot be entirely conscious. Typically, it occurs through the process of socialization, along with the development of consciousness itself. This process is different from communicating knowledge; rather, it correlates the individual development with the cultural environment.

Logic can never be formalized and described in full. However, there may be relatively stable schemes, since the cultural development assumes a sequence of distinct stages, historical epochs. This distinction depends on the level of generality, and a few stages may be merged in a single stage on a higher level, as well as any separate stage may be viewed as a succession of shorter historical periods. The corresponding regularities in logic can be studied with the scientific methods, and the logical norms can be enumerated and formalized.

Logical schemes

Every logical scheme can be treated as a structure, a system, or a hierarchy..

Structurally, a logical scheme contains a number of *logical positions* connected with *logical junctions*.

The structural aspect of logical schemes is used for *definition*. Every logical position is characterized by a unique collection of properties, and the process of categorization (which is the basis for analytic creativity in general) relates an empirically distinguished object to a position in some logical scheme. Inversely, the only definition on object can get comes from its relations to the other objects, which is reflected in an appropriate logical scheme.

As a system, the same logical scheme may describe a number of possible *inferences*.

The systemic aspect of a logical scheme implies its splitting into a number of substructures, so that any of them may be used to infer the rest of the scheme. The inferences that restore less missing parts may have a higher grade of certainty. However, no inference can be absolutely trustable, since the same substructures may participate in different schemes, either of which may be used for inference, producing different results. Hence, logical inference produces mere *hypotheses* which are yet to be checked in the practice.

From the hierarchical viewpoint, the scheme represents the levels and ways of logical development.

The scheme may be represented as a number of interrelated structures or interacting systems, forming a higher-order integrity. There may be more than two levels, and the resulting hierarchical structure or hierarchical system will represent one of the possible paths of development, from simpler schemes to the more complex ones. For logical hierarchies, the higher levels may be considered as more general, than the lower levels — levels of *generality*. However, development may follow different paths, and the same integrity may be constructed from different components.

Discreteness and continuity

Logical schemes reflect both continuity of activity and its divisibility into separate actions. The scheme is discrete since it contains a finite number of logical positions and junctions. However, both logical positions and logical junctions reflect continuity. The internal continuity of logical positions is different from the external continuity of logical junctions. Because of reflectivity, logical positions and junctions may interchange, internal continuity transforming into external, and vice versa.

The discrete aspect of logic reflects the objectively developed organization of activity. The two kinds of continuity correspond to the infinity of the ways leading to that level of development and the infinity of the ways of further development. The present is different from the past and the future, but it is not isolated from them either.

Scheme generation

Since creativity may be represented by certain activities, it may be creatively reflected, and this new level of reflection may be reflected too. Accordingly, there is the logic of activity, the logic of the analysis of that activity, as well as the logic of that analysis. Science reconstructs the logic of its subject, following its own logic, which is revealed in the methodological study.

Both as a component of any activity and as a special activity, logic can be reflected in art, science or philosophy. In particular, scheme generation obeys its own logic, which can be discovered and communicated.. Schemes may be either empirically found, or derived from the other schemes, or simply suggested for some general reasons. Actually, all the three ways are mixed in the development of logic. The derivation of schemes may be either integrative (constructing a new scheme from a number of other schemes) or differentiating (unfolding a scheme). One more important class of scheme generating operations is the transmutation of the logical positions and logical junctions, reflecting the mutual reflectivity of objects and their properties (or their environment).

Scheme generation is different from the process of inference, which may produce new constructions on the basis of a fixed logic only. However, the possible formal representations of scheme generation may look like logical inference, albeit on a higher level of generality. Due to the reflectivity and refoldability of hierarchies, there is no absolute distinction between inference and scheme generation.

The formal rules of scheme generation may be arbitrarily combined. However, the resulting schemes remain meaningless unless there is an activity obeying the corresponding logic. The practice may adopt a new scheme, or put it aside, so that it is either forgotten or rediscovered in the future, when there is more practical need for it.

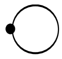
A few empirical rules of scheme generation:

- ***Negation*:** If there is **A**, there must be a **B** which is different from **A**. There may also be found some **C** different from **B**, and thus an infinite chain $\ldots \to \mathbf{A} \to \mathbf{B} \to \mathbf{C} \to \ldots$ is obtained. This is the *iterative infinity* (known in philosophy as "bad" infinity).
- If there is the sequence $\mathbf{A} \to \mathbf{B}$, there must be the sequence $\mathbf{B} \Rightarrow \mathbf{A}$, as well as the unordered pair $\mathbf{A}$ — $\mathbf{B}$, representing the chain $\ldots \to \mathbf{A} \to \mathbf{B} \Rightarrow \mathbf{A} \to \mathbf{B} \to \ldots$
- ***Differentiation*:** Every **A** and **B** are the representatives of the opposite classes: **[A]** and **[not A]**, or equally **[B]** and **[not B]**. For every two classes there are the *typical representatives* such that their difference represents the difference of the whole classes.
- ***Integration*:** Every **A** and **B** have something in common, which can be denoted as **C** distinct from both **A** and **B**.
- Though **A** may be a representative of a class **[A]**, **A** never coincides with **[A]**, and the class **[A]** may be considered as a representative of the class **[not A]**.
- ***Mediation*:** Every junction $\mathbf{A} \to \mathbf{B}$ may be associated with an **M** *performing* the junction, $\mathbf{A} \to \mathbf{M} \to \mathbf{B}$.
- ***Unfolding*:** If **A** relates to both **B** and **C**, then there are two different sides in **A** corresponding to its relations to **B** and **C** respectively. For instance, the scheme $\mathbf{A} \to \mathbf{B} \to \mathbf{C}$ leads to the scheme $\mathbf{A} \to (\mathbf{B}(\mathbf{A}) \to \mathbf{B}(\mathbf{C})) \to \mathbf{C}$. With "internal" mediation in **B**: $\mathbf{A} \to (\mathbf{B}(\mathbf{A}) \to \mathbf{B}(\mathbf{M}) \to \mathbf{B}(\mathbf{C})) \to \mathbf{C}$. Unfolding reflects the *potential infinity* of every object.
- ***Folding*:** Every scheme (or a part of a scheme) with *n* components may be represented by a single-component scheme, so that all the junctions of the original scheme become self-junctions in the folded form (reflectivity). This *monad* contains all the possible unfoldings — *actual infinity*. The combinations of folding and unfolding result in the *refoldings* of the scheme.
- If a scheme of an activity reproduces a logical scheme, the activity may be considered as either a prototype, or an application of the logical scheme. In particular, two logical schemes may be considered in parallel, as the versions of the same scheme.

There are numerous special versions of these rules adapted for particular applications. Due to reflectivity, they may become definitions and inferences, loosing their methodological orientation. Schemes of any level of complexity can be thus constructed.

Basic schemes

Logical schemes can be many-component and rather complicated — for example, modern mathematics as a scheme. However, there are a few simple schemes that may in many cases be considered as fundamental, so that the more unfolded schemes might be thought of as originating from these simple constructions.

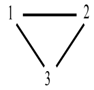
Monad. A single component joined to itself: . The only logical position of the monad is virtually identical to the only junction. Still, monads are not trivial, since they can be unfolded into any other scheme, with applying different sequences of generation rules. The structural aspect of the monad corresponds to the well-known principle of identity, reflecting the constancy of the subject of consideration (which defines the science studying it). On the systemic level, the monad states the component's transformation into itself, which, for example, describes the normal development of science that does not lead to contradictions requiring an extension of the limits of study. Hierarchically, the monad reflects the integrity of activity in the course of development: new levels may form, but they will be the levels of the same hierarchy.

Dyad. Two components linked to each other: **1** — **2** . The dyad represents the opposition of the components, their difference and mutual definition. There are two complementary unfoldings of the dyad: $\mathbf{1} \to \mathbf{2}$ ("primary", "material") and $\mathbf{2} \Rightarrow \mathbf{1}$ ("secondary", "ideal"). Thus, the primary form of the dyad may mean causation, while the secondary form will mean implication. The opposition of the primary and the secondary is the other side of the contrast of the logical positions **1** and **2**, and its particular meaning depends on the application.

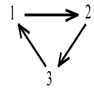
The mutual transition of the opposites in the dyad may be expressed as **1** ↔ **2**, or **1** ⇔ **2** , which may be interpreted as correspondence or equivalence. The forms of the dyad may be used to generate the schemes of any complexity. Dyadic reasoning is dominating in science.

The interaction between **1** and **2** may be viewed as a cycle of mutual determination: … **1** → **2** ⇒ **1'** → **2'** ⇒ … In this cycle, **1'** and **2'** are the different forms of 1 and 2 respectively, or the successive levels of their development.

Triad. Three components linked together:



, which implies two complementary cycles:



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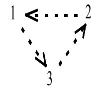
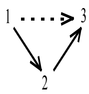


The most common unfolding of the triad is **1** → **2** → **3**, representing the *mediation* of the arrow **1** ⇒ **3** with the component **2**. The complementary (secondary) unfolding **3** ⇒ **2** ⇒ **1** is the mirror refection of the primary sequence. Hence, the component **2** represents the both arrows **1** ⇒ **3** and **3** → **1**, and virtually the dyad **1** — **3**. The same holds for any other pair of components and the respective arrows. So, the triad may be unfolded as (**1** ↔ **3**) → **2**, (**1** ↔ **2**) → **3**, or (**2** ↔ **3**) → **1**.

The dual forms of the triad:



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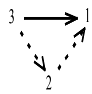
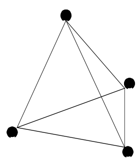


The arrow **1** ⇒ **3** is the *contraction* of the sequence **1** → **2** → **3**, and the arrow **3** → **1** is the contraction of the sequence **3** ⇒ **2** ⇒ **1**. This illustrates the difference and mutuality of the primary and secondary arrows. Once again, the components and arrows are interchangeable, and the arrow **1** — **3** synthesizes the arrows **1** — **2** and **2** — **3** just like the component **2** represents the dyad **1** — **3**. Evidently, the triad can be obtained from the dyad by unfolding the arrow and thus mediating the opposition.

Tetrad. The scheme with four components may be depicted as a tetrahedron:



The tetrad allows many unfoldings, including triad contractions like (**123**) → **4**, opposition of dyads like (**12**) ↔ (**34**), three-level unfoldings like (**12**) → **3** → **4**, and the complete unfoldings like **1** → **2** → **3** → **4**. Every such unfolding is a separate logical scheme, with its specific meaning. The complete tetrad as the synthesis of all the possible unfoldings is much more difficult to grasp than the monad, dyad or triad — and it is hard to discover in a real activity.

Of course, one might also conceive the schemes of the order higher than four, though such schemes seem completely impractical. Most many-component logical schemes can be reduced to the simplest combinations of the basic schemes, never producing an "*n*-ad".

Triads and development

The triad is the simplest unfolding of the dyad, and it can be used as a model of logic overcoming the traditional binary logic of scientific reasoning, including the idea of development. The triad stresses that the junction between any two components is a component of the scheme too.

In structures**:** the link between two elements is an element as well.

In systems**:** the transformation of one structure into another is a specific structure.

In hierarchies**:** the ordering of two levels is a level of the same hierarchy.

Triads are most convenient for unfolding, since every one of its components is joined to two others. So, the components of the triad are easily made hierarchies, both potentially and actually infinite. Contraction of the triad to a point produces a form of tetrad, with the distinction of the contracted and unfolded levels.

The genesis of the triad:

1. All the components merged together: (**123**). This means that there are random (unstable) distinctions due to some lower-order mechanisms.
2. The distinction of the components **1** and **3**, with the *external* (to the triad) links between them: **3** → **1**. This looks like an *internal* process **1** ⇒ **3**.
3. *Interiorization* of the external links: **1** → **2** → **3**, with the mediation **2** as an actual internal process transforming **1** into **3**, the higher-order mechanism of **1** ⇒ **3**.
4. Reproduction of the triad: … → (**1** → **2** → **3**) → (**1'** → **2'** → **3'**) → … It involves higher-order external links **3** → **1'**. As a result of the folding (interiorization) of this link, the internal processes **3** ⇒ **2** and **2** ⇒ **1** are formed.
5. The two internal cycles (**1** → **2** → **3**) and (**3** ⇒ **2** ⇒ **1**) get contracted in a higher-order activity.

This is the usual way of development in the world, including its physical (existential) level, the level of life, and the level of activity and reason. However, this sequence may become inverted in reflection, and the logic of communicating the results may be different from the logic of obtaining them.

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