Qualitative Complexity

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Abstract

Different attitudes to the problem of complexity evaluation are categorized within a general scheme representing the levels of integrity. Complexity becomes distinguished from singularity and unity, and implies three distinct types: multiplicity, coherence and order. Structure, system and hierarchy are described as the levels of coherence, and the respective measures of complexity are discussed. The principles of hierarchical approach are formulated, and its relations to the structural and systemic approaches are traced.

Introduction

Contemporary science has come to a clear understanding of the necessity of studying the development of any object to comprehend its structure and behavior [1]. The first intuitive idea of an object's development associates it with the growth of the object's complexity and the existence of different levels within the object. The consideration of hierarchical structures and hierarchical systems [2] leads to the natural question: "What is the multilevel organization as such, and where it comes from?" In particular, such investigation might bring light to the problem of the distinction of structural and systemic description, which often get mixed in the literature [3]. One more goal that might be achieved in such a study is the reconstruction of the object's integrity and discovering the directions of its development, rather than focusing on arbitrary details and the peculiarities of behavior [1].

The recent interest to the study of complex systems poses, along with the numerous technical issues, many fundamental questions. What is the organization of a system? How "complex" systems differ from "simple" ones? What should be meant under the "small", "large" and "superlarge" system? Thus, the sense of complexity may differ at different levels and for different objects — and it would be desirable to have some qualitative distinctions before trying to construct any formal measures of complexity. Indeed, there are many such quantitative measures [4,5], and it is not always clear what they actually evaluate.

In many cases, the answers are sought in hierarchical considerations. The literature is replete with different hierarchical constructions, and some authors suggest distinguishing the hierarchy (development) as a separate level of complexity, along with the structure (static level) and the system (dynamic level) [1]. It is well known that hierarchical organization may be the key to efficient control in large systems [6]. Yet another hint comes from nonlinear physics, considering strongly non-equilibrium systems with their own order. It has been found that the behavior of such systems may be dependent on

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the sequence of catastrophes they have passed to the moment of observation, and hence the history of the system would be represented in its current state [1,7]. Development and hierarchical organization were thus related to nonlinearity, self-action.

Analogous phenomena have been known in philosophy since long ago, under the name of *reflection*, which, however, was mostly associated with subjectivity. The recognition of the different forms of reflection might lead to a better understanding of the differences between the physical, organic and subjective levels of development. This would be one more step to the integrity of science, which has been earlier sought within structural and systemic approach [8].

Levels of integrity

Each science is dealing with its specific object, and any consideration has to start from the fundamental concepts which cannot be introduced within the science and should be borrowed from somewhere else, representing the first intuitive view of the field. On the other side, these *a priori* concepts outline the scope of relevant problems and delimit the range of applicability. For studying complexity, the notion of *integrity* might be taken for such a starting point. Thus, to speak about complexity of something, one must first insure that this "something" may be considered as an entity distinguished enough from the rest of the world. That is, the natural premise to complexity studies is the existence of integral "wholes" whose complexity could be further described. This means that complexity may only be defined in respect to that integrity, being one aspect, or one form of it.

The first, most primitive form of integrity is *singularity*. At this level, the object is considered as unique and isolated, without any regard for other objects. No internal organization or external relations are considered, and therefore the object is quite *simple*. The only definiteness it may possess is its very existence. Not much can be inferred from such a primitive consideration — still, this is the necessary first stage of any study, the recognition of the problem.

On the next level, the simplicity of this recognition gives place to the observation of external dependencies and internal inhomogeneity — this is where one can speak about *complexity*. The object is considered together with its environment, and the object's interaction with it leads to specific structures, processes or kinds of development. The object is no longer unique and simple, being rich enough to be studied by various sciences, from their specific points of view. The integrity of the object may therefore seem violated, being potential rather than actual, and a "metascientific" approach is required to provide a unified view.

The level of *unity* restores the singularity of the object retaining its complexity. The object becomes completely reflected in its environment, while this environment is completely represented "inside" the object. The features of the object are just the traces of its history, and its behavior is non-local, being controlled by the higher-level development. However, this level escapes purely scientific consideration, being essentially influenced by practice.

Now, when the level of complexity has been related to the other levels of integrity, one can proceed with unfolding the hierarchy, distinguishing different types of complexity itself. Its definition as a path from singularity to unity provides a logical basis for such a distinction. Thus, one can conclude that there are two sides of complexity reflecting its relation to these extremes, and that there should be an intermediate level linking them into a hierarchical whole.

The level of complexity extending singularity in a minimal way is *multiplicity*. There are many instances of the same singularity, as if produced by some cloning procedure, when each clone remains simple and isolated from the others, but not unique this time: there are many such objects, defining a specific object class by the very fact of their existence. Still, the objects of the same class enter no interrelations beyond the simple equivalence, mere belonging to the same class. Any one of them could be chosen as a representative of that class, and the whole class can be restored from every single element. Therefore, the complexity of such class may be related to the number of its elements, and the hierarchy of multiplicity coincides with the hierarchy of cardinal numbers.

The unity side of complexity might be called *order*, including both the sense of "being properly made, arranged", and the sense of "as it should be". In a sense, this is the "most complex" complexity, since it cannot be comprehended in the purely objective terms, being unfolded into a "teleological" hierarchy. Historically, the difference between multiplicity and order is the ancient opposition of Chaos and Cosmos — the opposition that gave birth to all the earthly things. This earthly way from Chaos (multiplicity) to Cosmos (order) is the intermediate level of complexity introducing some congruencies into the chaotic multiplicity, while leaving enough space for extensive and intensive development of the local order. This kind of complexity might be called *coherence*.

So, multiplicity is associated with disorder, coherence means partial order, while on the highest level order becomes complete, universal.

Moving deeper into the hierarchy of complexity, one could use the same logical scheme, distinguishing the opposite aspects of coherence joined by an intermediate level. This procedure leads to the three levels of coherence: structure, system and hierarchy.

The first category of this triad, *structure*, refers to internal coherence, representing the object as a collection of elements and their links. This representation is least different from multiplicity, the only new feature being the division of the multiplicity into two classes, one called "elements" and the other called "links". Being the internal characteristic of the object, structure may be thought of as the *static* aspect of the object.

The inverse of structure is *system*, the second level of coherence. It refers primarily to the external manifestations of the object, the way it "moves" in its outer space, altering its relations with the environment. Since these relations are somehow structured, system may be generally considered as the way of transforming one structure into another. So, the basic category at the systemic level is "transformation", or "transition" — and therefore system represents the object's *dynamics*.

Integrity

Singularity → Complexity → Unity

Multiplicity (Aggregation) → Coherence (Combination) → Order (Construction)

Structure → System → Hierarchy

Figure 1. The hierarchy of integrity.

Logically, the next level of coherence should be the synthesis of the internal description provided by structure and the external systemic treatment. It should consider the object both statically and dynamically, so that systemic transformations lead to the internal changes in the object, which nevertheless retains some of its structural features as to remain *the same* in these transformations. This is the level of *development* — and the synthesis of structural and systemic features is *hierarchy*.

Thus, complexity itself becomes complex, comprising the hierarchy of possible forms (Fig. 1). One level of distinction provides the triad of multiplicity, coherence and order — on another level, one might distinguish structural complexity, systemic (functional) complexity, and hierarchical (developmental) complexity. Incidentally, this sequence reflects the history of methodological thought in the XX century: the beginning of the century was marked by the structural approach, which gave way to systemic approach in the middle of the century, while the end of the XX century passed under the dominance of the idea of development, which receives its formal expression in the hierarchical approach.

Structure

The most general idea of structure is linking some relatively distinct elements by a number of links. Typically, structure is modeled with a set and relations on it: the elements of the set represent the elements of the structure, while the links are associated with the *n*-tuples of the elements belonging to an *n*-place relation. However, the links may be treated as independent entities, like arrows in the categorial approach [9]; in this case, one needs to explicitly define the beginning and the end of each arrow. The support set may be either discrete, or continuous, or even more powerful. Accordingly, the relations may vary from the finite number of element pairs to connectivities on a non-trivial manifold. Links may be either rigid, or stochastic, or any combination of the two. All these possibilities fall under the scope of traditional mathematics, which may be called the science about structures, in general. Since structure refers to the static side of the whole, it becomes clear that mathematics is incompatible with any motion, and this explains why mathematicians made their best to expel movement (and development) from mathematical language, and even the modernistic mathematical trends (like constructivism) speak of dynamics in a static way, imposed by the traditional forms of mathematical reasoning. That is, the mathematical description of a process refers to the *structure of the process* only; accordingly, mathematical models of development mainly reflect its structural aspect.

The simplest structure is given by a finite set $S = \{s_i: i = 1,\ldots,N\}$ with a single two-place relation Λ: $S\rightarrow S$ defined on it. When a pair $<s_i, s_k>$ belongs to relation Λ, one says that element s_i is linked to element s_k by the link $\lambda_{ik} \in \Lambda$. Such link is oriented, and $\lambda_{ik} \neq \lambda_{ki}$; moreover, relation Λ need not contain both λ_{ik} and λ_{ki} , so that if one element is linked to another it does not imply that there must be a link back. Denoting the set of the elements of S which are linked to some other elements with dom(S) and the set of the elements that appear in the right-side of the pairs from Λ with rng(S) , one can observe that, in general, dom(S) $\neq$ rng(S), dom(S) $\neq S$ and rng(S) $\neq S$. In the trivial case, Λ is empty, and the structure reduces to mere multiplicity. At the opposite extreme, any element is directly linked to any other, and $\Lambda = S^2$.

However, structure is more than just elements and links — it is a kind of wholeness, a level in the hierarchy of integrity. In the above model, the appearance of this integrity might be described as follows.

The direct links between the elements of S represented by $\lambda \in \Lambda$ are not the only connections between them. Thus, the relation Λ may contain both pairs λ_{ik} and λ_{km} , which means that there is a *mediated* link between s_i and s_m (Fig. 2a) — and this does not depend on whether there is the direct link λ_{im} or not.

Longer chains may be constructed as well, and one comes to considering the hierarchy of indirect links which is one more manifestation of the same structure.

Yet another structural feature is the formation of *collateral* links. For example, if λ_{ik} and λ_{mk} both belong to relation Λ, elements s_i and s_m are naturally related to each other as the predecessors of the same element (Fig. 2b). Similarly, if $\lambda_{ki} \in \Lambda$ and $\lambda_{km} \in \Lambda$ then there is a collateral link between s_i and s_m , which have a common predecessor (Fig. 2c).

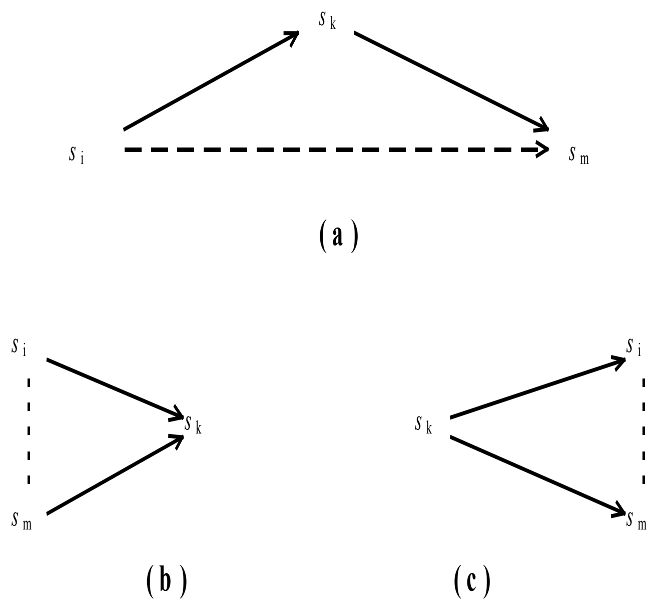

Figure 2. Indirect links: (a) mediated; (b,c) collateral.

Direct, mediated and collateral links may be combined in various ways, the numerous kinds of *indirect* links thus obtained being the manifestations of the same structure. If an element $s \in S$ participates in at least one pair $\lambda \in \Lambda$, it becomes, in one way or another, connected with any other such element. The elements which are not linked to any other element (or to themselves) by Λ are completely irrelevant to the structure, so that the set dom(S) $\cap$ rng(S) can be considered as the set of the structure's elements in the applications, instead of S. Note the difference between *irrelevant* and *isolated* elements: the former merely do not belong to the structure, while the latter are just linked to themselves only, with no direct or indirect link to any other element.

The distinction between elements and links within the structure may be relative. Thus, if element s_k mediates the link between elements s_i and s_m , it may be considered as a higher-level link connecting λ_{ik} and λ_{km} . Since any two elements of the structure (discarding the irrelevant elements) are somehow connected, any element can thus become a link between links, so that the links will play the role of the structure's elements. Hence, any particular subdivision of the structure into elements and links does not follow from its own properties, but rather from some conditions external to the structural approach proper. When a number of "primary" elements and links are selected, the rest of the structure can be accordingly unfolded; for another choice, the structure will unfold differently. Such refoldability makes the structure *hierarchical*.

The existence of different unfoldings, with the respective levels of integrity, means that there is no universal quantitative measure of *structural complexity*. Moreover, even though one might evaluate structural complexity for every particular unfolding, there may be a hierarchy of different measures, not always reducible to a single number. Thus, in the simple relational model described above, one might count the total number of links and divide it by N^2 (the maximum possible) to obtain a kind of probability (frequency) p. Then, a global measure of structural complexity could be introduced as

$$I_0 = -p \log p - (1-p)\log(1-p),$$

which is the well-known formula for the quantity of information. The value I_0 is equal to zero when there are no relevant elements in the structure, or for a maximally connected structure, when $\Lambda = S^2$. This agrees with the intuitive idea of structural complexity: the structures without links are quite simple, as well as the "rigid" structures with the elements linked in a "completely deterministic" way.

An alternative approach is to count the number of "arrows" beginning at a given element s_k and divide it by N to obtain the normalized values p_k lying in [0,1]. Evidently,

$$p = \frac{1}{N}\sum p_k ,$$

so that the "probability" p introduced via counting links is just the *average* "probability" of an element being linked to the structure. Since all the p_k are mutually independent, one could evaluate the information contained in the set $\{ p_k\}$ as

$$I^{(+)} = \sum I_k^{(+)} = -\sum\left(p_k \log p_k + (1-p_k)\log(1-p_k)\right).$$

Analogously, one could define the value

$$I^{(-)} = \sum I_k^{(-)} = -\sum\left(q_k \log q_k + (1-q_k)\log(1-q_k)\right),$$

where q_k are the counts of arrows with the end at the element s_k divided by N. Though, evidently, the average frequency p may be expressed through q_k too as

$$p = \frac{1}{N}\sum q_k ,$$

the quantities $I^{(+)}$ and $I^{(-)}$ do not coincide, and the measure I_0 becomes split into two dual measures $I^{(+)}$ and $I^{(-)}$.

Of course, the process can be continued, to account for indirect links and substructures. For example, every two elements s_i and s_k may be assigned with a numerical weight c_{ik} indicating the "level of connectedness" of these elements within a given unfolding of the structure. The weights c_{ik} can be chosen from the interval [0,1] so that $c_{ik} = 1$ if the two elements are connected in every possible (direct or indirect) way, while $c_{ik} = 0$ would mean that there is no connection between the elements, that is, the structure splits into mutually isolated substructures. Then, a gross measure of complexity can be introduced as

$$I = -\sum_{i,k=1}^{N} c_{ik}\log(c_{ik}).$$

The set of weights $\{c_{ik}\}$ may be considered as a fuzzy subset of S^2 [10,11]. In general, c_{ik} cannot be interpreted as probabilities, since they do not necessarily satisfy the "normalization conditions", as specified in [11]. However, there may be classes of valuation functions that can be associated with cumulative probability distributions [12]; the complexity measure I will become a kind of entropy in this case.

I should stress that structure as a level of coherence does not imply any restrictions on the type of elements and links. Thus, there may be “material” structures, with both elements and links of a material nature; however, there may also be completely “ideal” structures, or some mixtures of the two.

System

A typical abstraction of system might be represented by a collection of triads $\{<S_{in}, S_c, S_{out}>\}$, where S_{in} and S_{out} are the input and output structures respectively, while S_c denotes the current state of the system, often identified with its “internal” structure. Depending on the level of consideration, each of these three structures may be differently unfolded, providing the special models known in the literature. Thus, the completely folded S_c leads to the notion of “black box”, which evidently correlates with the idea of elementary operation in the theory of computability [4], or with the basic arrows in the categorial approach [9]. In a more unfolded form, S_c may be any composition of such elementary operations, implementing an algorithm of “calculating” the output structure by the input structure, the “white box” [13]. Complexity on the systemic level may therefore be called *algorithmic*, or *computational complexity* [4].

One might develop a simple model of system analogous to the relational model of structure described in the previous section. Thus, S_{in} and S_{out} might be chosen from the same class of structures representing the states of the system’s *environment*; then they will be analogous to the elements of the structure, while operators S_c connecting them will be the analogs of structural links λ. The only difference is that the “elements” connected by such functional link are *external* to the system, unlike internal elements of the structure. This is the characteristic duality of any system: on one side, it functions like a structured object — while on the other side it can be considered as just a more detailed specification of a structural link.

The formation of mediated links finds its systemic-level analog in the external *composition* of systems, when the initial state S_i of the environment is transformed into the final state S_f via an intermediate state S^*:

$$S_i \xrightarrow{S_{1c}} S^* \xrightarrow{S_{2c}} S_f\,,$$

which may be considered as the construction of a new operator $S_c = S_{2c} \circ S_{1c}$. Like with structures, such *sequential* compositions (or *cascades*) can form long chains; since an elementary systemic transformation (operation) may be thought of as a *transition*, the composite functions represent *processes*. For example, the movement of a point $\boldsymbol{x}$ in a configuration space $\boldsymbol{X}$ can be considered as sequential transformation of structures:

$$\ldots \to (\boldsymbol{x}, t) \to (\boldsymbol{x}', t') \to (\boldsymbol{x}'', t'') \to \ldots$$

In this case, the operators transforming one structure into another must be associated with the respective elements of the tangent space $T\boldsymbol{X}$, velocities. Such an approach is typical for classical physics, and especially classical mechanics.

The other kind of indirect links, collateral links, can be associated with the *parallel* composition of systems, when several input or output structures are united into a joint input/output. This means that a *class of structures* would serve as the system’s input or output, instead of a single structure; along with the basic structures, such class would include all the possible sets composed of the basic structures. For example, a binary input is a single-element structure s ; when two such structures s and s' are composed into a parallel input, there may be combinations (s), (s') and (s, s') as the possible values of the same input. In a more complex case, one could consider some distributions of elementary inputs as the "microscopic" realizations of a "macroscopic" variable. Such parallel composition of systems is widely employed in statistical physics. Various combinations of sequential and parallel composition may be found, for instance, in quantum theory.

The external nature of systemic coherence leads to a kind of integrity quite different from the internal integrity of the structure. The system's integrity has to be comprehended from all the variety of its relations with the environment, rather than from the internal structure of the system. Generally, functional complexity is revealed dynamically, in the process of functioning [14]. Consequently, it cannot be described in a static way, and this is the main source of any problems with "computability", leading to the numerous forms of the famous Gödel theorem [15].

Systemic complexity is complexity of functioning, and it should not necessarily correlate with the complexity of the structures involved. Functional complexity is the property of a single element, or a structure as a whole, rather than of the way the elements are connected, and, in this sense, it is *complementary* to structural complexity [16]. For example, a computer program may be *very* long — but all it does is a constant output; a nail may be driven in either with a hammer, or using a complex cybernetic device, etc.

However, the complexity of the "white boxes" modeling a system would generally correlate with functional complexity if these models are built of some "standard" elements, whose functional complexity does not change when they are connected into a system. In the simplest case, the *external* model of a system ("white box") may be constructed of the elements of unit complexity — and then the algorithmic complexity of the composite system would be represented by the complexity of the junctions. Such systems are completely "transparent", though they do not have to be deterministic.

Still, there is a difference between the system and its model of the "white box" type. Since the goal of such modeling is to reconstruct functioning only, the model may be built of the blocks different from the "matter" of the original system — and this would allow a partial reconstruction of behavior only, with some properties of the original system discarded. That is, the original system is modeled on a definite level — and the variety of such models is the systemic counterpart for the various unfoldings of the structure. Usually, all the lower-level functioning is considered as *side effect*, so that different systems model each other to that accuracy. However, there is also an analog of the structure's refoldability: the properties that are considered as side effect in one situation, may be essential in another.

Like the distinction of elements and links of the structure may be relative, there is a mutability of subsystems and their junctions. Thus, for the sequential composition of two functional blocks described above, the triad $<S_{1c}, S^*, S_{2c}>$ may be considered as a component of a system, so that the intermediate structure S^* will play the role of the internal structure of this system, rather than the state of environment. In the operator $S_{2c} \circ S_{1c}$, the junction $\circ$ (represented by the structure S^*) transforms the output of S_{1c} into the input of S_{2c}.

As in the case of structures, systems may be either material, or ideal, or of a mixed type. The definitions of this section remain applicable in each case — though the functional treatment of the system might be not evident sometimes. Thus, systematization often means mere classification, which seems to be closer to the structural level. However, taxonomy can be a system if it is used for categorization, implementing the transition from the appearance of the object to its essence, and then to its more subtle features. Still, there is no rigid boundary between the structural and systemic levels, and they usually become intricately interwoven in practice, representing the two sides of the object's hierarchy.

Hierarchy

Though hierarchical approach may be considered as a logical completion of the historical line from the structural methodology to the system paradigm, the notion of hierarchy is much older, ascending to the mythological cosmology of the primitive societies. The first manifestation of hierarchy is the presence of several qualitatively different levels with a kind of vertical order, when one level may dominate over another, so that the relations between the levels are of a kind other than the relations inside each level. Up to the recent time, the origin of this order was unknown — and hence hierarchy seemed to be imposed by some supreme force, which is reflected in the very word "hierarchy": "the sacred order". Now, it is clear that the levels of hierarchy represent the stages of its history, and that *reflection* (nonlinearity) is the key to any development [1].

Most generally, reflection is the interaction of the object with itself, which implies self-relation and self-transformation. At the structural level, reflection can be represented by linking an element of the structure to itself; in particular, the reflexivity of a relation $\Lambda: S \to S$ means that $<s, s> \in \Lambda$ for any element $s \in S$. However, this is not the only way to introduce reflection into the structural description, since an element of the structure may be linked to itself indirectly, via mediated or collateral links. The depth of indirection may be a criterion for the distinction of the different levels of the structure, when it is unfolded starting from a fixed element. Of course, the same structure may be unfolded in many such *hierarchical* structures.

For the system, reflection is easily associated with a cyclic process, when the system's output may change its environment, which would affect the system's input, and so on; this is the common feed-back scheme. When the part of environment that provides such feed-back is included into the system, the system acquires at least two levels, one of which corresponds to the "pure" functioning, while the other accounts for "self-regulation", like in the usual operation analysis [13, ch. 4]. The system thus becomes *hierarchical*.

Since any hierarchy can only manifest itself through the variety of its hierarchical structures and systems, there may often be a lack of awareness of the hierarchy itself. The different structural and systemic description then seem uncorrelated and even controversial, and there may be hot argument between their adepts, claiming their own attitude the only truth. However, these contradictions are most likely to be merely apparent, being the aspects of the integral description [17].

The basic features of hierarchy might be summarized as follows:

* Hierarchy can be unfolded into numerous hierarchical structures, and its external behavior is, at any instance, that of a hierarchical system.
* There are no rigid levels of hierarchy, but rather hierarchy is characterized by infinite divisibility. Thus, the relations between any two levels of hierarchy constitute a specific entity which may be considered as a level of the same hierarchy lying *between* the two original levels. Therefore, there is no "complete" structure of the hierarchy, since one can always find a new level between any two previously discovered.

* The collection of intermediate levels between any two levels of hierarchy may be *folded* into their direct connection, so that the total number of levels would be diminished. The different ways of folding and unfolding the hierarchy lead to its various manifestations, or *refoldings*.
* Because of refoldability, there is no absolute "topmost level" in the hierarchy, though any hierarchical structure would possess one. Any element of hierarchy may become its top unit, thus representing the hierarchy as a whole.
* Hierarchy is not a simple ordering of levels, but rather a multidimensional formation. The number of its dimensions is as infinite, as the number of levels. However, each unfolding implies a one-dimensional ordering of levels, and the levels may be characterized by a definite dimension.
* Within hierarchy, the distinction between the elements and their connections may only refer to a single unfolding, thus being relative. In the same way, any functional decomposition is related to a definite hierarchical system, based on the respective unfolding of the hierarchy.
* There is a kind of self-conformity in the hierarchy. Any component of hierarchy is a hierarchy too, and it may be unfolded in the same way as the whole hierarchy. The very distinction between the part and the whole becomes relative, since every single element of hierarchy reflects it all, contains it within, thus being equivalent to it.

The "own" hierarchy of any object is another side of the hierarchy of its environment. Reflexive interaction with the environment leads to the object's development. Since refoldability assumes many ways of interfacing the external world, development may follow different routes, and different unfoldings of a hierarchy indicate the possible ways of its development. Being the unity of the internal and the external, hierarchy assumes two directions of development: it may either "zoom in" unfolding its elements and their connections — or it may grow through joining several hierarchies in one. These acts of integration and differentiation change the *organization* of hierarchy.

Like with the indirect links in the structure, or the processes at the systemic level, the interactions of the objects in the world may be mediated by other objects, up to the most distant influences. The integrity hence arising unites the objects with their environment, making the whole world a unity. However, this unity should be treated hierarchically, and it cannot be comprehended as a given entity, or a process — again, it is a synthesis of the both.

The object's interaction with the world may be represented by the cycle of alternating phases (levels) of action and being acted upon. The object is *reproduced* in each cycle, though in another state. The simplest case of such reproduction is hierarchical refolding, leaving the object the same and merely changing its "form", or its "position" in the world. One more possibility is *extensive* reproduction, or *expansion*, when a larger part of the world becomes involved in the object's environment, while the character of interaction remains generally unchanged. The next level is *intensive* reproduction, or development proper, which implies a shift of the boundary between the object and its surroundings, the change in the very notion of the internal. Evidently, this means a synthesis with some other hierarchy, formerly attributed to the external world.

One cycle of the object's self-reproduction provides a natural measure of *time*, associated with this particular development. Such time should be considered as hierarchy, since the cycle of reproduction looks differently at different levels of hierarchy, thus defining different time "scales". It differs from the time variable known in physical sciences, where it is a structural parameter rather than a measure of the level of development, *hierarchical complexity*. The hierarchical notion of time reflects its intuitive features, such as directedness from the past to the future, the existence of a finite "now" within each reflection cycle, and the difference in the "natural" time flow for the objects of different type.

Conclusions

The hierarchy of integrity discussed in this paper may be unfolded in different ways. One of them has lead to the hierarchical understanding of complexity, which could become a framework for further qualitative and quantitative specifications. Like structure, or system, the category of hierarchy is *universal*, so that any object can be treated hierarchically. All the hierarchies are identical in their organization, and may be considered the unfoldings of the same hierarchy, the different sides of the same world. This may pose many delicate questions concerning the correspondence between natural or artificial hierarchies. Thus, ideal links may become quite material bonds, directedness of development may assume the form of purposefulness, the abstractions of scientific analysis and synthesis may transform into practical development as destruction and reconstruction. One could further unfold the hierarchy of complexity, to cover the categories like "collection", "arrangement", "compound" or "mixture". Another direction of unfolding leads to such characteristics as "balance", "stationarity", "stability", "robustness" etc. One of the most important areas of hierarchical study is the investigation of different levels of mediation: passive, random mediation is typical for the inorganic world, while the organic level is characterized by active, or forced mediation, and the level of subjectivity is marked by the universal and arbitrary mediation, when any two objects become interrelated due to the projection of the world into the mind.

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12. H. M. Hubey, "Special classes of fuzzy truth valuations and probability theory" (1996); online: *http://www.csam.montclair.edu/~hubey/papers.html*

13. N. Wiener, *Cybernetics or Control and communication in the animal and the machine* (New York, London: MIT Press, Wiley, 1961)

14. M. D. Mesarovic and Y. Takahara, *General systems theory: mathematical foundations* (New York, San Francisco, London: Academic Press, 1975)

15. P. B. Ivanov, "Computability in developing systems" (1996); online: *http://unism.narod.ru/arc/1996cd/cde.htm*

16. M. Mesarovic, "General systems theory and its mathematical foundations" *IEEE Transactions on Systems Science and Cybernetics*, vol. **SSC-4** (1968)

17. H. M. Hubey, "Intelligence, regression, multiplicativity, path integrals, and stochastic models of scaling and evolution" (1996); online: *http://www.csam.montclair.edu/~hubey/papers.html*

More texts:
http://unism.narod.ru/arc/index.htm

Comments

Below, find a few remarks made on this paper by the reviewers of *Complexity International*, followed by my replies.

Reviewer 1:

In the section dedicated to “Structure”, before the first formulae, there is the idea that the mathematical description only allows for a static view of the objects described. I would relativize that issue, since mathematical descriptions can well include within their own formulation a possible evolution of the model (see logics of action, temporal logics, or some categorical treatments in theoretical computer science).

Reply:

To indicate my acquaintance with such ideas, I should probably have made a reference to some book containing an overview of the related branches of mathematics — *e. g*. D. A. Pospelov, *Situation-driven control: Theory and practice* (Moscow: Nauka, 1986). I am not inclined to relativizing the issue, since, up to now, any attempts of incorporating time (movement, functioning, evolution) in mathematics were entirely structural, as far as I can judge. Today, mathematical reasoning is poorly suited to discuss dynamics, or development; the present trend towards ever more abstract formality indicates that functional and developmental ideas can hardly receive an adequate expression in the mathematics of the nearest future. Additional considerations on the subject could be found in: P. B. Ivanov, *Computability in developing systems* (1996).

There is yet another aspect, the common prejudice about the essence of science. Traditionally, a scientist is supposed to treat a very narrow and special subject, and any generalizations are to be presented as if they had been made on the basis of the bulk of such special investigations. But the true logic of scientific research is exactly the opposite: any special research is always regulated by some general considerations which determine the choice of subject, the methods of research and the ways of interpreting the results. There is no science without such a conceptual background, and the attempts to disguise this important circumstance by a primitive inductive style are mere relics of a centuries-old philosophy (F. Bacon *etc*.), slightly renewed by the logical positivism of the XX century. I suppose that it’s high time to abandon that stylish primitivism in order to make scientific papers more logical, and indicating the place of any special choice in a wider picture. In particular, the arrangement of the results might better reflect the natural direction of activity, from general ideas to their implementation.

Reviewer 1:

The following use of mathematical notation is rather awkward, and a much more elegant presentation could be done using category theory for instance (however I doubt it would really be necessary, when one looks at what is achieved using this pseudomathematical model: mere introduction of the notion of structural complexity as an entropy-like function).

Reply:

Complexity International started as a methodological journal, trying to synthesize the variety of applied research into an integral view of complexity in general. My paper was an attempt to revive this orientation, which has eventually become dissolved in the rush of highly technical papers of the last volumes. Here, mathematics serves as a mere illustration of general ideas, and I didn't intend to obtain any formal results, which is stressed by the very title of the paper. Simple notions are better suited for illustration, so that I don't have to waste two thirds of the text explaining the notation. Yes, the category theory is an amusing toy; but it develops in the same conceptual frame as the rest of mathematics, and brings no principal novelty in the discussion of fundamental problems.

I would stress once again that structural complexity is not a number, since any numerical estimates of complexity belong to a different level, namely, that of *multiplicity*. Structural complexity can be represented by a hierarchy of numerical measures of complexity, but it can never be reduced to any specific measure. Rather, different structures might be used as the units of structural complexity, just like numbers measure multiplicity.

Reviewer 1:

In the section on "System", there is a reference to Gödel, which could be skipped. The idea that functional complexity is only revealed dynamically and that a mathematical description is a priori static, is not very closely related to the incompleteness theorems.

Reply:

I agree, that the relation between the incompleteness theorem and the insufficiency of the structural approach of mathematics should be additionally clarified. In this paper, I express my opinion without a due substantiation, which is far from the current style of "scientific" reports. According to the present norms, I should first have publish a paper on the Gödel theorem, and then quote it in context of this discussion. Unfortunately, there is logical circularity: such a paper on incompleteness would necessarily involve the ideas expressed in the present paper, which should hence have been presented beforehand… A phrase might well be skipped, to please the reviewer; still, I'd rather leave it as it stands, since the paper will not be published anyway.

Reviewer 1:

I enjoyed the multiple foldings involved in the section on "Hierarchy", it is an interesting idea of the whole paper.

Reply:

Thanks. This is a very important feature of hierarchies, with numerous examples in the literature, albeit without a clear awareness.

Reviewer 1:

The conclusion is very speculative, and personally I do not share such dream visions. I would greatly recommend a rewriting of that conclusion.

Reply:

Sorry, but I hate the style of concluding sections that just list the "main results" of the paper, duplicating the abstract or/and the introduction. For some papers, that could do; but only for those in the traditional line, pretending to establishing final truths. A piece of work that has not yet been (and, in fact, can never be) finished should better conclude by indicating the ways of further development, or suggesting additional conceptual links to think over.

Reviewer 2:

English: a lot of mistakes (about 16 on the front page only)

Reply:

Since English has become the language of international communication in science, there is a flavor of language chauvinism in academic journals: native English-speakers are more likely to pass the barrier, and the references to non-English sources are discouraged; in any way, reviewers now have a quick excuse for rejecting a paper without going into any particulars. Everybody knows that efficient communication does not require perfect phrasing, or correct spelling: all the misunderstandings can be fixed in the flow of communication. Those interested in the ideas would not waste time counting language lapses.

Reviewer 2:

A paradigm is discussed which defines complexity in terms of "integrity", "structure" and "system". Part of the discussion is mathematical, the rest being merely philosophical. Despite very interesting ideas, its content is often inaccurate (for instance: "Structure is more than just elements and links, it is a kind of wholeness, a level in the hierarchy of integrity") and conclusions are hazy ("Functional complexity leads to the numerous forms of Gödel's theorem"). The math parts are a bit basic.

Reply:

Well, I have to admit it. The discussion is mainly methodological, with minimum mathematics, just for illustration. It would be nice to learn which particular ideas have attracted the reviewer's attention. Still, I do not find any inaccurateness in the quoted sentence on the essence of structure; in fact, it is much more accurate than the usual mathematical definitions, reducing structure as such to arbitrary special models.

Reviewer 2:

My advise would be to read "Chaos and Information Theory: an heuristic outline", Nicolis and Prigogine, World Scientific, 1990.

Reply:

I am well acquainted with the Prigogine's line (and I reference one of Prigogine's books in the paper). My approach is quite different; though, of course, I appreciate the value of Prigogine's ideas for the comprehension of the necessity of incorporating development in science. However, I suppose that his theory deals with only one of the possible kinds of development , and that it is insufficient in other cases (especially, in social sciences). This is a topic for a special discussion.

With all that, why should I discuss somebody else's views in my text instead of those my own? And why should my views merely expand somebody's ideas, rather than follow my own way of thought?

Reviewer 3:

While the subject of making explicit various aspects of the notion of complexity is an appealing one, I think that the paper lacks precision in its use of terms, and fails to present a logical and rigorous argument from well defined contentions to conclusions.

Reply:

The principal goal of my paper was to clarify the ideas related to complexity, and for that purpose, I suggest a new way of definition, relating any notions to their hierarchical context. The reviewer has missed that point, because of the common prejudice that the only precision possible is that of syllogistic deduction. However, neither deduction is the only source of rigor, nor can it be rigorous enough, as my previous paper, *Computability in developing systems*, clearly indicates The standard form of discourse imposed by "scientific" journals is nothing but a tribute to an obsolete tradition.

Reviewer 3:

In a paper of this kind one would expect to find:

1. In the introduction, a clear statement of the research question being tackled including how the research builds on the existing body of research. I would expect to find reference to specific papers (rather than whole volume citations) defining the point of departure of the research, what has gone before and what new methods are to be employed. There is a substantial literature concerning the definition of complexity.

2. In the body, clear definitions of terms, clear descriptions of analytical models and methods to be used to make the argument, and an economy of description demonstrating a clearly worked out argument.

3. A clear conclusion summarising the contribution of the research and remaining open questions.

Reply:

This is a good summary of bad style. The numerous articles like that pretend that they really contribute into development of science, while timidly hiding any valuable thought in the haze of references to the predecessors. Certainly, the text should be as clear as possible. However, following the above formal requirements would rather obfuscate any problem.

An overview of the previous work in the same field adds nothing to the contents of the paper, while increasing its size and obscuring the author's intentions. Such historical issues should rather be treated in a special appendix, or even in a separate paper devoted specifically to the history of science.

Problems that have not yet been studied to any considerable extent cannot have a clear preliminary formulation. A concise exposition of the principal line of research would restrict the problem to a particular approach, which may prove utterly inadequate in the end. A general indication of the scope is quite enough for an introduction, since it is the whole body of the paper that is to exactly specify the issues to consider. Scientific research differs from engineering in that the latter starts with a (more or less comprehensive) list of features to implement, while science is bound to ramble in the dark to discover phenomena yet unknown.

"Precise" definitions are only possible within a very narrow portion of research, where nothing really new is expected to come, beyond a rearrangement of the already available. Activities like that may be of some use for certain pragmatic purposes, but they have nothing to do with science. Likewise, the methods to employ will normally crystallize in the course of their employment, and not before real research. To demonstrate a clearly worked and economical argument, one needs to entirely solve the problem, which is impossible in any serious study. Such "sharpened" descriptions appear much later, as simplified accounts, in the educational context.

A summary of the author's contribution in the conclusion would mean that a final solution is already at hand, which is almost never the case. The only honest kind of summary is to remind what the author *did* (but never *have done*) in the body of text; still, isn't it what an abstract is for? The author can never enumerate the *remaining* open questions, just because there is an infinity of such questions, including those considered in the text. Of course, the author is free to speak of his personal interests and priorities; the reader may however be interested in anything else.

Reviewer 3:

Instead we find many undefined terms which have precise meaning only for the author, a set theoretical model that is introduced and then largely abandoned, and a rambling development which includes a number of irrelevant metaphysical speculations.

Reply:

The reviewer did not pay attention to the specific type of definition employed in this work: the categories mutually define each other through being used in the same context, in different positions. This method of definition is no less precise than the traditional reduction to something previously introduced (and which always needs to be somehow defined in its turn). An illustration of some particular viewpoint (say, a set theoretical model of structure) may happen to be much less useful to illustrate a different idea; one does not need to stick to a single picture throughout the whole. "Metaphysical speculations" (methodological research) are absolutely necessary to avoid blind technicality, purposeless manipulation with empty symbols and arbitrary terminology.

Reviewer 3:

The paper demonstrates a lack of understanding of the incremental nature of scientific investigations and the degree of precision required for publication in a scientific journal.

Reply:

The reviewer demonstrates a lack of understanding of the impossibility of incremental evolution in science, and the inevitability of scientific revolutions. He confuses science with engineering, or mere craftsmanship, denying any real creativity.

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