The effects of relative phase and the number of components on residue pitch

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The pitch of the residue produced by 6- and 12-component waves whose components were either in cosine or random phase was measured as a function of the frequency region of the components. The components were all of equal amplitude and the frequency spacing between them was 200 Hz. Waves with the same lowest component were found to produce the same pitch: neither the number of components nor the relative phase of the components was important in determining the pitch of the residue. If every component in a wave was a multiple of 200 Hz, the wave produced a 200 Hz pitch; if such a set of components was shifted in frequency by a small amount, there was a corresponding linear shift in the pitch of the residue. When the lowest component in the wave was below about 900 Hz, the slope values associated with the lines relating the pitch shift to the frequency shift varied little from their average value of 0.21. As the lowest component increased from 900 to 2580 Hz the slope values decreased to about 0.08. These findings are in good agreement with the current model of pitch perception based on interpeak durations.

Subject Classification 4.9.

INTRODUCTION

A. The Residue

A pitch that is produced by a set of frequency components rather than by a single sinusoid is called a residue (Schouten, 1940). For example, any set of three or more adjacent harmonics of 200 Hz produces the same pitch as a 200-Hz sinusoid provided the components are below about 4000 Hz (Ritsma, 1962). The timbre associated with this residue pitch varies, in general becoming sharper as the center frequency of the set of component sinusoids increases; however, the pitch remains constant. Figure 1(a) is a schematic representation of the power spectrum of a 12-component residue-producing stimulus. It gives rise to a 200-Hz pitch despite the absence of power at that frequency. When the components are all in cosine phase the corresponding waveform is that pictured in Fig. 2(a).

B. The Pitch Shift of the Residue

When a set of adjacent harmonics that produces a residue is frequency-shifted by up to one-half the distance between components, there is a corresponding, monotonic shift in the pitch of the residue. For example, as the set of 12 components in Fig. 1(a) is shifted up in frequency by 60 Hz to the position shown in Fig. 1(b), the associated residue pitch rises from 200 to 209 Hz. Figure 2(b) is the waveform produced by the power spectrum in Fig. 1(b) when all the components are in cosine phase. The pitch shift of the residue was first noted by Schouten in 1940, but it received little attention until 1956 when de Boer carried out the first major study of the phenomenon. De Boer used waves composed of five sinusoids spaced 200 Hz apart. He showed that the pitch shift is a linear function of the frequency shift. In addition he showed that as the center frequency of the complex increases, the propor-

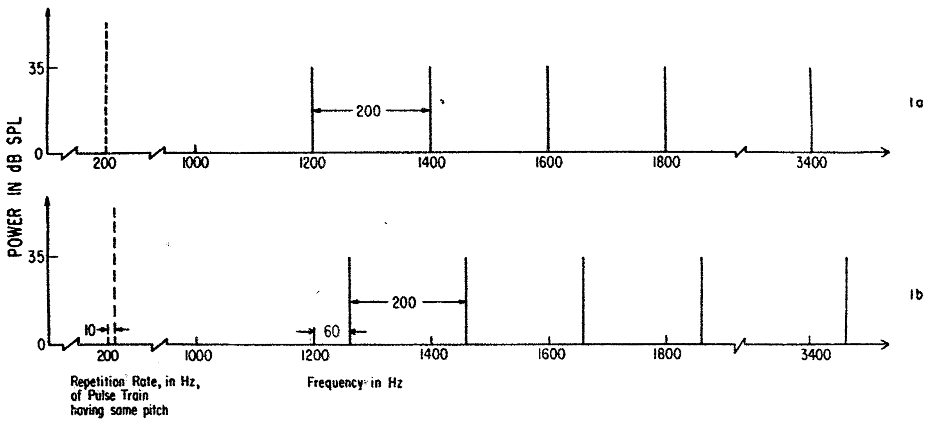

FIG. 1. Schematic representation of the power spectra of two 12-component residue-producing stimuli with frequency shifts of (a) 0 and (b) 60 Hz.

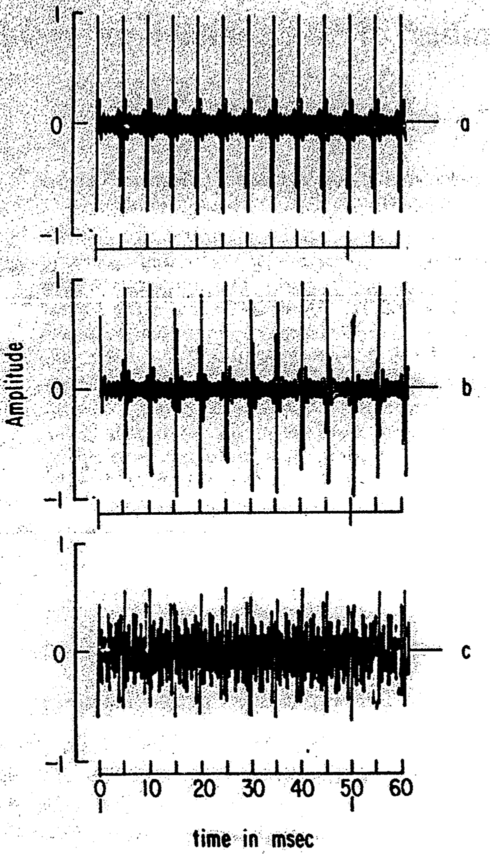

FIG. 2. Oscilloscope photographs of three 12-component residue-producing waves: a and b are the waves associated with the spectra in Figs. 1(a) and 1(b) when the components are all in cosine phase; c has the same power spectrum as b—however, the starting phase of each component is assigned at random.

tionality constant relating the pitch shift to the frequency shift decreases. Schouten, Ritsma, and Cardozo (1962) measured the pitch shift of the residue for three-component waves and confirmed de Boer's findings. Thus, in the data of Schouten *et al.*, a three-component wave whose lowest component was 1260 Hz produced a 209-Hz residue, and one whose lowest component was 1860 Hz produced a 206-Hz residue even though the two waves have the same frequency shift.

I. THE PRESENT EXPERIMENT

A. Number of Components

De Boer (1956) observed that three waves which had the same center frequency (1660 Hz) and the same frequency shift (60 Hz), but a different number of components (five, nine, or 15) produced slightly different pitches; pitch increased with the number of components. In the present experiment, the pitch of six- and 12-component waves is measured and the results are combined with those of Schouten *et al.* (1962), who used three-component waves to provide a detailed description of the relationship between pitch and number of components.

Recently, Bilsen and Ritsma (1969–1970) have argued, convincingly, that the pitch of the residue is determined by energy in the region around the fourth harmonic of that pitch whenever there is audible energy in that region. And Walliser (1969) has provided evidence that the residue pitch produced by a set of high harmonics is determined by the lowest component. That section of a wave's spectrum which determines pitch can be estimated from a comparison of the pitches produced by waves with different numbers of components. For example, two three-component waves whose lowest components are 1260 and 1860 Hz produce pitches of 209 and 206 Hz, respectively (Schouten *et al.*, 1962). Compare these pitches with the pitch produced by a six-component wave whose lowest component is 1260 Hz. The six-component wave has the same lowest component, 1260 Hz, as the three-component wave that produced the 209-Hz pitch and the same highest component, 2260 Hz, as the wave that produced the 206-Hz pitch. If the pitch of the residue is determined by the lowest component of the complex, the six-component wave will produce a 209-Hz residue; if it is determined by the highest, it will produce a 206-Hz residue. The results are compared with the conclusions drawn by Bilsen and Ritsma (1969–1970) and Walliser (1969).

B. Relative Phase

Mathes and Miller (1947) showed that the relative phases of the components of a residue-producing wave affect the tonality or timbre of the residue. A set of three equally spaced sinusoids, 900, 1000, and 1100 Hz, in which all the components are in phase, produces a much stronger and rougher residue pitch than the same set of sinusoids in which the phase of the center component has been shifted by 90°. Similarly, Licklider (1957) reported that a set of eight equally spaced tones produces a clear residue when the components are all in phase but that the residue is markedly reduced when the phases of the components are adjusted haphazardly.

While it is generally agreed that relative phase can affect the tonality of the residue, it is not clear what effect relative phase has on the pitch of the residue. In de Boer's (1956) study of the pitch shift of the residue, most of the data were gathered with five-component waves in which all components were in cosine phase. De Boer does report, however, that he filtered some of the waves to produce different phase relations and found it did not affect the pitch of the residue. Similarly, Smoorenburg (1970) reports that the residue pitch of two tone complexes is not affected by changes in the phase relation between the two components. On the other hand, Ritsma and Engel (1964) found that the pitch of the residue produced by a three-component wave was altered by shifting the

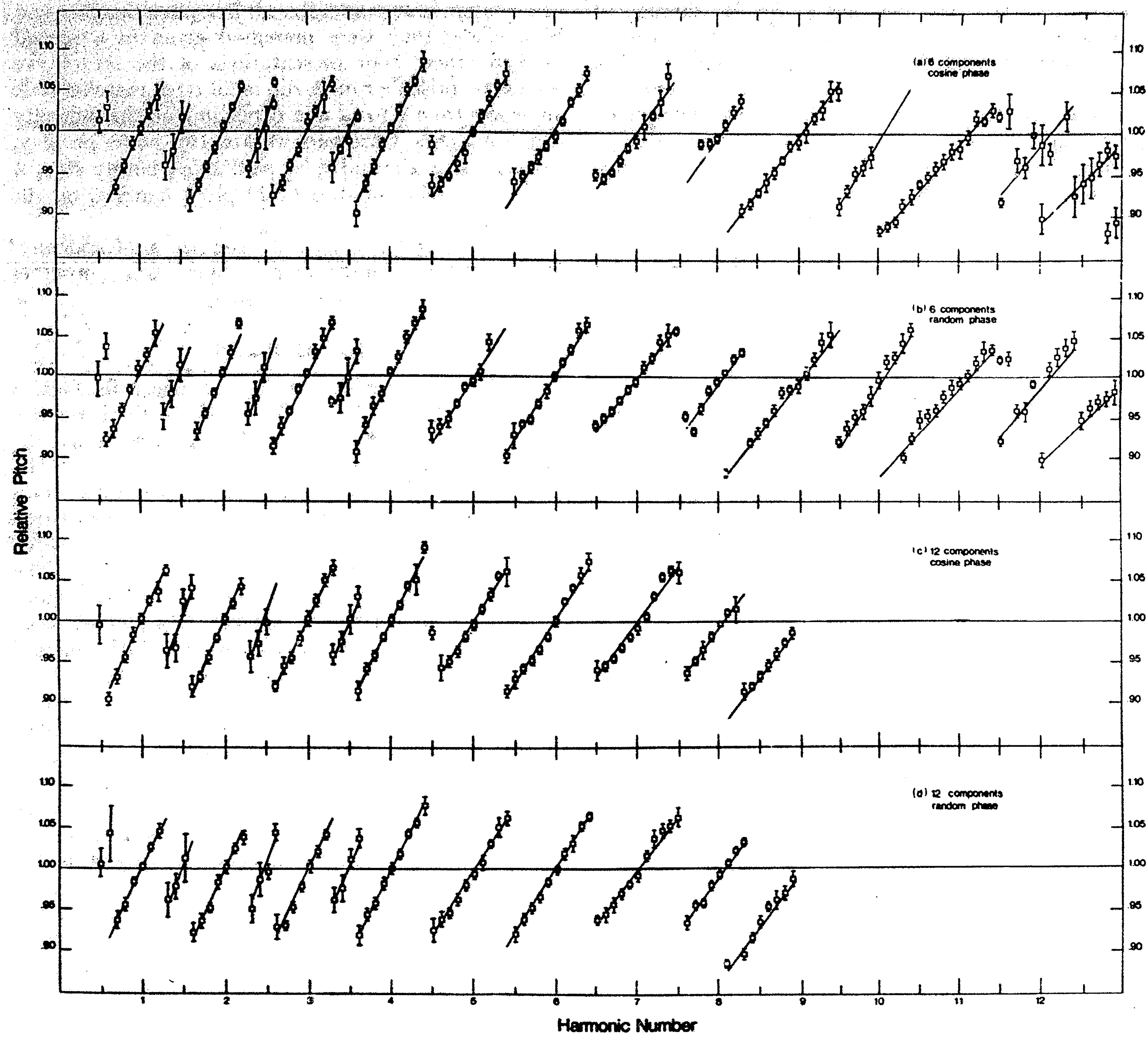

FIG. 3. Mean pitch-match data for one observer, DS, as a function of the frequency of the lowest test-wave component, the number of components in the test-wave, and the relative phase among the components. The error bars are ± one standard deviation.

phase of the center component. When the three equally spaced components were in phase, the wave produced one relatively strong pitch. However, when the center component was shifted 90°, the wave gave rise to two rather weak pitches, one above and one below the pitch produced in the in-phase case.

In an effort to determine whether relative phase affects the pitch of six- and 12-component waves, we compared two relative phase conditions. In one case, the components were all in cosine phase. In the other case, the starting phase of each component in the wave was assigned a value between 0 and 2π radians at random. Figure 2(c) is a picture of a random-phase wave. It has the same power spectrum as the wave pictured in Fig. 2(b). Only their phase spectra differ. The results show that the two phase conditions invariably produced the same pitch despite marked differences in wave shape.

II. METHOD

The experiment is conceptually very simple. A test wave whose pitch we wished to determine was presented to an observer who adjusted the pitch of a comparison stimulus to produce a match. The pitch of the comparison stimulus was known and provided a measure of the pitch of the test wave.

The test wave was a set of either six or 12 sinusoidal components. The frequency difference between adjacent components was held constant at 200 Hz throughout the experiment; all components in a wave were of equal amplitude (40 dB SPL). In a particular stimulus, either the components all started in cosine phase or the starting phase of each component was determined at random. Therefore, a particular test wave can be uniquely identified by specifying the frequency of one of the components, the number of components (six or 12), and the relative phase among the components (cosine or random). A test wave will be described in terms of its lowest component, which in this experiment ranged from 100 to 2600 Hz in 20-Hz steps.

The comparison stimulus was a bandpass-filtered pulse train. A pulse train was used because it has a timbre roughly similar to that of the test stimuli. The pitch of the pulse train is given by its repetition rate, i.e., a sinusoid with a frequency of f Hz and a pulse train with a repetition rate of f pulses per second have the same pitch, although the pulse train has a much raspier timbre. To further reduce timbre differences, the pulse train was filtered (Krohn–Hite model 330N); only energy in approximately the same region of the spectrum as that occupied by the test stimulus was passed. Tests were conducted to insure that filtering the pulse train did not affect its pitch.

The test waves were produced by digital rather than analog means. Test-wave amplitude was calculated, at $\frac{1}{10}$-msec intervals, by first computing the amplitude of each component sinusoid at that instant and summing the results. The completed waves were stored in a computer and played, at a 10 000 points/sec rate, into a 12-bit digital-to-analog converter, the output of which was filtered to improve the off-band attenuation. There were no suprathreshold distortion products. Every frequency component in every test wave was a harmonic of 20 Hz. The period of a 20-Hz sinusoid is 50 msec. All harmonics of 20 Hz are strictly periodic in 50 msec. Thus, it was only necessary to calculate and store a 50-msec segment of each test wave, as that segment can be continuously repeated to produce any test-wave duration.

The experiment was run by a small computer (PDP-8). An observer was presented a 500-msec sample of a test wave followed by a 500-msec sample of the comparison wave. If he pressed the leftmost of his two response buttons the pitch of the comparison wave was lowered (the period of the pulse train was increased) and if he pressed the rightmost button the reverse occurred. If he did not respond within four seconds of the presentation of the comparison wave, the pair of samples was repeated. The observer was instructed to raise and lower the pitch of the comparison wave until it matched that of the test wave. If he heard two or more pitches in the test wave, he was to match the most obvious one.

There was only room to store five test waves in the computer. The waves were presented to the observer in a random order and, after all five pitch matches had been made, they were presented again in a second random order. Four presentations of the set of five waves comprised a run. A run of 20 pitch matches took anywhere from 6 to 12 min depending on the difficulty of the matches. Observers worked two hours per day, five days a week, and were paid at an hourly rate. A two-hour session produced 120 pitch matches on the average.

Observer JL had played trumpet professionally; observer DS was an amateur trumpet player; observer MH had very little musical training.

III. RESULTS

The data from the experiment appear in Figs. 3 and 4. The abscissa, harmonic number, is the frequency of the lowest test-wave component divided by the frequency separation between components, 200 Hz. The ordinate is the repetition rate of the pulse train that has the same pitch as the test wave. Repetition rate is also measured relative to the frequency separation between test-wave components, 200 Hz. The ordinate is a tenfold expansion of the abscissa. Figure 3 shows the data for one observer, DS, plotted separately for the different combinations of "relative phase" and "number of components". The error bars about each mean are plus and minus one standard deviation. Figure 4(b) shows the results for DS when we collapse over the "number of components" and "relative phase" variables. Thus, the point above "8" in Fig. 3(c) is the mean of the pitch matches made by DS in response to the stimulus whose 12 components all start in cosine phase and whose lowest-frequency component is the eighth harmonic of 200 Hz; whereas, the point above "8" in Fig. 4(b) is the mean of all of the pitch matches that DS made in response to the four waves whose lowest component is the eighth harmonic of 200 Hz, independent of the number of components in the complex or the starting phase of those components. The other two sections of Fig. 4 show the results for observers JL and MH. Figure 4 represents more than 14 000 individual pitch matches (JL, 3400; DS, 4000; MH, 6600). Observer JL did not have time to complete the experiment and as a result there are no data for him in the regions 1400–1580, 1800–1980, and 2200–2380 Hz. The lines drawn through the points in Fig. 4 were fit to the data using a least-squares criterion.

A. Major Results

The lines in Fig. 4(b) were traced onto each section of Fig. 3 to facilitate comparison of the different experimental conditions in the case of observer DS. If there were any treatment effects, they would appear as systematic deviations from these overall regression lines. Comparison of Fig. 3(a) with Fig. 3(b) and Fig. 3(c) with Fig. 3(d) reveals that randomizing the

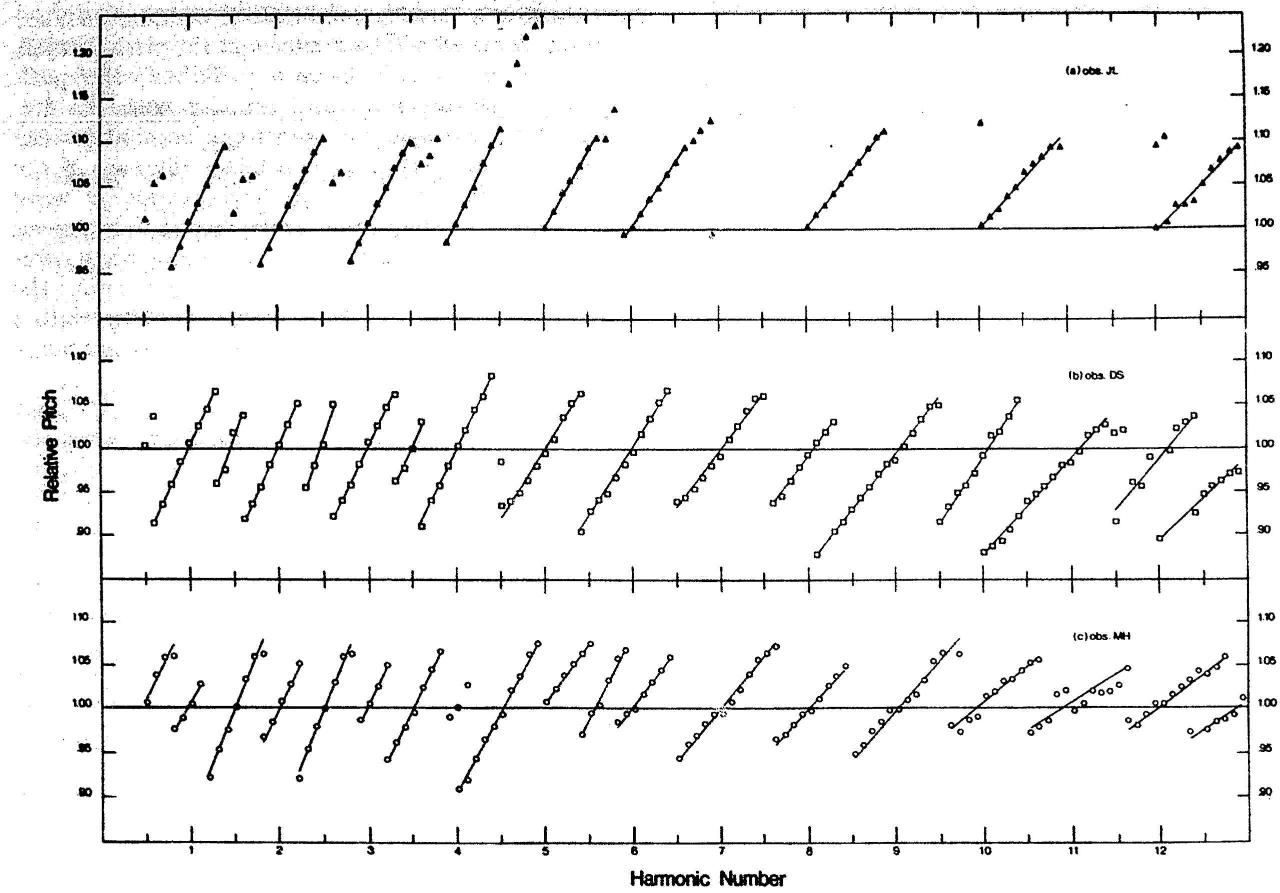

FIG. 4. Mean pitch match for three observers, JL, DS, and MH, as a function of the lowest test-wave component.

starting phase of the individual component sinusoids does not affect the pitch of the residue for this observer despite the enormous effect that it has on the shape of the waveform [compare Figs. 2(b) and 2(c)].[1] The overall regression lines almost always pass between the error bars about the means. Comparison of Fig. 3(a) with Fig. 3(c) and Fig. 3(b) with Fig. 3(d) shows that for this observer the pitch shift of the residue-producing wave does not depend on the number of components (six or 12) used to produce the residue. Further comparison of these data with those of de Boer (1956) and Schouten *et al.* (1962) reveals that the conclusion can be generalized to both five- and three-component waves. The data from the other two observers lead to the same conclusion: in these experiments, residue-producing waves with the same lowest component produce the same pitch independent of the number of components in the complex or the relative phase among those components.

B. Ambiguity of Pitch

Typically, the distribution of pitch matches made by an observer in response to a particular test wave was unimodal and quite peaked. However, sometimes an observer heard two pitches in a test wave and, as a result, produced a bimodal distribution of matches in response to repeated presentations of the wave. Schouten *et al.* (1962) found that many of their stimuli gave rise to two or even three distinct pitches. In their experiment, at the start of each set of matches, they instructed the observer as to which pitch to match. In the present experiment, an observer was told to match to the most obvious pitch. Whereas their procedure reveals the different possible matches, our procedure provides some measure of the relative strength of the pitches.

If we look at a particular subject's data in Fig. 4, we find that, for the most part, only one mean occurs in conjunction with each lowest-component frequency, indicating that, for a particular observer, one pitch typically dominates in the perception of a given wave.[2] However, when we look at the means above one abscissa value for different observers we find an interaction: different pitches dominate for different observers. For JL high pitches tend to dominate; for DS, at least at larger abscissa values, low pitches are relatively stronger; for MH there is an even balance. Although the reason for the differences in relative pitch strength between observers is not apparent, the differences are

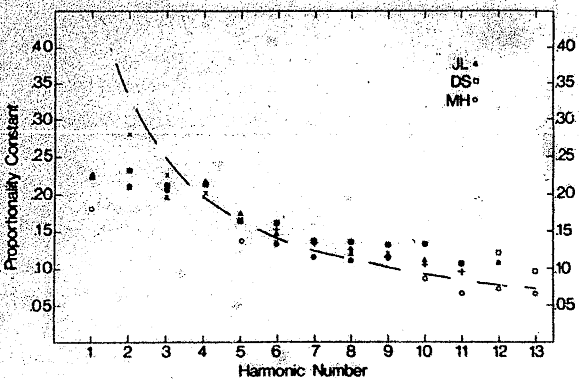

FIG. 5. Slopes of the lines in Fig. 4, plotted as a function of harmonic number. The dashed line shows the slope values that are predicted by the models of de Boer (1956), and Schouten *et al.* (1962) where the effective center frequency for a wave is assumed to be its second lowest component.

reliable and stable over time. The data summarized in Fig. 4 were gathered over a one-year period. Replications of experimental conditions were often separated by three months or more, yet the individual differences shown in Fig. 4 appear consistently in each replication.

C. Proportionality Constant

The slopes of the regression lines of Fig. 4 are plotted in Fig. 5 as a function of harmonic number and observer. Solid symbols are used in those cases where the regression line accounts for greater than 90% of the variance. (The 95% confidence limits associated with the solid symbols are about ±0.005.) The "×" and "+" symbols show the results obtained by de Boer (1956) and Schouten *et al.* (1962), respectively.[3] De Boer and Schouten *et al.* plot their data as a function of the center component of the residue-producing wave. In the present paper, the data are plotted as a function of the lowest component, since it was found that waves with the same lowest component produce the same pitch. In Fig. 5, the slope values from all three experiments have been plotted as a function of the lowest component, and under this condition the results from the different experiments are found to be in remarkably good agreement.[4]

IV. DISCUSSION

De Boer (1956) and Schouten *et al.* (1962) have described a model of pitch perception that can accommodate their two basic findings: (1) that the pitch shift of the residue is a linear function of the frequency shift of the components, and (2) that the slope of the line relating pitch and frequency shifts decreases as harmonic number increases. In their description of the model they suggest that the hearing mechanism measures the time between successive major peaks in the wave and uses the reciprocal of this duration as a measure of the wave's pitch. If every frequency component in a wave is a multiple of 200 Hz, then the interpeak duration will be 5 msec and the pitch 200 Hz. When such a wave is frequency shifted, interpeak duration varies inversely and linearly with the frequency shift, and thus the model can account for the finding that the pitch shift is a linear function of the frequency shift. Other things being equal, a wave composed of high-frequency components has interpeak durations that are closer to 5 msec than a wave composed of low-frequency components. Consequently, the model can also predict that the slope of the line relating pitch to frequency shift should decrease as harmonic number increases.

The model as described is qualitatively but not quantitatively correct. De Boer showed that when the components of the wave are all of equal amplitude, the model predicts that the slope of the line relating pitch to frequency shift will be $1/n$ where n, an integer, is the harmonic number associated with the center-frequency component. De Boer found, however, that his experimentally-determined slopes were always greater than these predicted values. Similarly, the regression lines derived from the six- and 12-component data have slopes that are greater than the model predicts. For example, the 12-component waves that have lowest components in the 1200-Hz range produce a line with a slope of about 0.15. But the center component (either 2200 or 2400 Hz) has a harmonic number of either 11 or 12, and consequently, the model predicts a slope between 0.091 and 0.083. To account for the discrepancy he found, de Boer proposed that the hearing mechanism low-pass filters the wave prior to measuring interpeak durations. Reducing the relative amplitude of the higher components reduces the center frequency which in turn increases the predicted slope values. The dashed line in Fig. 5 indicates the slope values that would be anticipated if the effective center frequency for these waves were the second to the lowest component:

$$\text{Slope} = 1/(\text{Harmonic Number} + 1).$$

The dashed line fits the data from all three experiments well in the range above the fourth harmonic. Thus, working within the framework of the theory, we may conclude that the second to lowest component dominates in the determination of residue pitch when all of the stimulus energy is above the fourth harmonic of that pitch.

When there is stimulus energy in the frequency region below the fourth harmonic, the slope values are much lower than the theory would predict. This result is not surprising in light of the recent series of experiments conducted by Ritsma and Bilsen (Bilsen and Ritsma, 1967–1968; Bilsen and Ritsma, 1969–1970; Ritsma, 1967; Ritsma and Bilsen, 1970). All of these experiments demonstrate that the pitch of the residue is determined by stimulus energy in the third to fifth harmonic region when such energy is audible. In the present experiment all waves with lowest components at or below the fifth harmonic have energy in this area. The experimentally determined slope values associated with these waves are fairly constant. The mean slope value for lines at and below the fourth harmonic is 0.21. This implies that the dominant frequency region is between the fourth and fifth harmonics (4.7) which compares favorably with Bilsen and Ritsma's (1969–1970) estimate of 3.9.

One further aspect of the data deserves discussion. Every test wave whose lowest component is a multiple of 100 Hz is entirely made up of components that are multiples of 100 Hz and, consequently, each of these waves has a 10-msec period. Thus, they might be expected to produce 100-Hz pitches but, in fact, no 100-Hz matches occurred. The pitch matches made to these waves were of two types. When the lowest component in the wave was a fairly high harmonic of 100 Hz, the wave was analyzed as a set of harmonics of 200 Hz that had been frequency shifted by 100 Hz. For example, when observer DS was presented with a wave whose lowest component was 1700 Hz [Harmonic Number=8.5 in Fig. 4(b)] he heard a 186-Hz pitch (Relative Pitch=0.93). This pitch match falls on the line of matches passing through the point (1.0, 9.0). However, when the lowest component was a low harmonic of 100 Hz, an extra line of pitch matches was obtained, a line that passes through the pitch axis (Relative Pitch=1.0) at a multiple of 100 rather than 200 Hz. For example, when observer DS was presented a wave whose lowest component was 500 Hz [Harmonic Number=2.5 in Fig. 4(b)] he matched the pitch produced with the pitch of a 200-Hz pulse train. And when this set of components was shifted in frequency by a small amount, the pitch shifted proportionately. Observers DS and MH produced these extra lines of matches provided the lowest component in the wave was below 720 and 1180 Hz, respectively. Observer JL showed few matches of this type. Both Flanagan and Guttman (1960a, 1960b) and de Boer (1956) report pitch match data on waves made up of the odd harmonics of 100 Hz in which low harmonics were present. Flanagan and Guttman report matches near 200 Hz to waves with 10-msec periods, but de Boer shows only data like that of observer JL.

One possible explanation for these extra lines of pitch matches, within the framework of the present theories, is that perhaps the observers (DS and MH) were making an octave error and that the test waves were actually being processed as sets of harmonics of 100 Hz. In other words, perhaps the hearing mechanism was extracting the pitch information from interpeak durations on the order of 10 rather than 5 msec. If it is assumed that interpeak durations around 10 msec are the basis for these pitch matches, then it should be possible to predict the slopes of these extra lines; that is, the slope of a line through an odd harmonic of 100 Hz, for example 500 Hz, should be twice the slope of the line through the same odd harmonic of 200 Hz, or 1000 Hz. Observers DS and MH show slopes of 0.29 and 0.25 at 500 Hz and slopes of 0.16 and 0.14 at 1000 Hz, which supports this hypothesis. In general, this system for predicting the slopes of the lines at odd harmonics of 100 Hz works for harmonic values greater than or equal to 5, as can be seen in Table I. However, below this value the slopes of the odd harmonic lines are not as steep as the hypothesis requires.

TABLE I. Slope values of lines associated with odd multiples of 100 and 200 Hz.

Harmonic number	Frequency of the lowest component	Observer	
		DS	MH
1	100		0.206
	200		0.180
3	300	0.285	0.252
	600	0.211	0.204
5	500	0.290	0.245
	1000	0.163	0.138
7	700	0.223	0.208
	1400	0.135	0.113
9	900		0.189
	1800		0.112
11	1100		0.197
	2200		0.064

To test the idea that observers were making octave errors, we compared test waves with lowest components of 100, 300, and 500 Hz alternately with 100- and 200-Hz pulse trains. All of these stimuli had the same tone-chroma;[5] however, the 200-Hz pulse train seemed to be an octave above the test waves while the 100-Hz pulse train seemed to be an octave below the test waves. Thus the test waves appear to have a tone-height that is between 100 and 200 Hz. The observers, then, were not making an octave error; rather, the octave of the test waves is ambiguous. Our data show no 100-Hz matches, but this is probably a direct result of our procedure, since at the start of each match the repetition rate for the pulse train was chosen at random from the range 160 to 250 Hz, which in all likelihood biased the observers towards the 200-Hz choice.

V. CONCLUSIONS

Residue-producing waves with from three to 12 components produce the same pitch provided they have the same lowest component (the components are of equal amplitude and are spaced 200 Hz apart); the pitch is not dependent on the number of components in the wave, and the pitch does not change when the relative phase of the components is randomized.

As the lowest component in the residue-producing wave rises from 100 to around 900 Hz, the slope values associated with the lines relating pitch shift to frequency shift do not vary appreciably from their average value of 0.21. This slope constancy supports the view of Bilsen and Ritsma (1969–1970) that the pitch of the residue is determined by the fourth harmonic region when the energy in this region is audible. As the lowest component continues upward from 900 to 2580 Hz, the slope values decrease to about 0.08. The same kind of decrease is found in the experiments of de Boer (1956) and Schouten *et al.* (1962) as in the present experiment. Within the framework of the theory of pitch perception proposed by de Boer and Schouten *et al.*, which is based on interpeak durations, all of these data suggest that it is the second to the lowest component that dominates in the production of pitch when the components in the wave are all above the fourth harmonic region.

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