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ENNEADECAPHONIC MUSIC. A NEW SYSTEM OF HARMONIC TONES.

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RIASSUNTO. — Si propongono ai Compositori le regole e i primi esempi di composizioni per strumenti a tasti accordati secondo un nuovo sistema di toni.

ZUSAMMENFASSUNG. — Eine kurze Einführung in die Theorie der Komposition in einem neuen harmonischen System mit 12 Tönen, die aus 19 gleichentfernten Tönen gewählt werden.

ABSTRACT. — A new system of harmonic frequencies is presented. Beethoven's themes are shown to give rise to samples of music for the new kind of scale. An example of new music, which could *not* be played with instruments tuned in the traditional dodecaphonic way, is given.

1. - J. S. Bach's « well » tempered dodecaphonic scale.

A first approximation for most basic « natural tones » (i.e. tones whose frequency quotiens are fractions with small integer factors both in the numerator and in the denominator) by means of a « tempered scale » (i.e. sequence of frequencies in geometric progression in order

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to be invariant under « modulation ») which despite its limitations has been tolerated so far, is given by the dodecaphonic tones suggested by J. S. Bach; indeed the powers of $\sqrt[12]{2}$ have approximately the values:

do	1	(unison)
do # = re b	17/16	half tone.
re	9/8	second.
re # = mi b	19/16	« compromise » between the *very* dissonant augmented second and the *very* consonant minor third which should be 6/5.
mi	24/19	major third which should be 5/4.
fa	4/3	fourth.
fa # = sol b	17/12	« compromise » between the *differently* resolvent augmented fourth and diminished fifth.
sol	3/2	fifth.
sol # = la b	19/12	minor sixth.
la	32/19	major sixth.
la # = si b	16/9	minor seventh.
si	32/17	major seventh.

The inconveniences of this classical dodecaphonic scale, sometimes pointed out by musicians, are due to the following three fundamental reasons:

(1) Among the above intervals just the « second » (9/8) and the « fifth » (3/2) and their inversions « minor seventh » (16/9) and « fourth » (4/3) actually manage to be enough consonant (while minor third and major third *should* also play fundamental roles in chords).

(2) On the other hand the « augmented second » does not manage to be enough dissonant (as it should be).

(3) Finally the « diminished fifth » (17/12) turns out to be very weakly consonant while it should play a rather nice role in the very common chord of dominant seventh with omitted fifth and therefore should not be considered dissonant.

These inconveniences are solved by the enneadecaphonic scale presented in this paper.

2. - The enneadecaphonic general scale.

A better approximation for most basic natural tones by means of a new tempered scale is here suggested first of all in a version of just theoretical interest. Each « octave » in this « enneadecaphonic general scale » should contain 19 tones whose frequencies grow in proportion to the powers of $\sqrt[19]{2}$, which have approximately the values:

do	= **1** =	(unison)	H. CONSONANT
do #	(28/27)	chromatic interval	dissonant atonal
re b	14/13	diminished second	R. C.
re	10/9	second	R. C.
re #	(15/13)	augmented second	dissonant atonal
mi b	**6/5**	minor third	V. CONS.
mi	**5/4**	major third	V. CONS.
mi # = fa b	9/7	diminished fourth	R. C. atonal
fa	= **4/3** =	fourth	H. CONSONANT
fa #	18/13	augmented fourth	R. C.
sol b	13/9	diminished fifth	R. C.
sol	= **3/2** =	fifth	H. CONSONANT
sol #	14/9	augmented fifth	R. C. atonal
la b	**8/5**	minor sixth	V. CONS.
la	**5/3**	major sixth	V. CONS.
la #	(26/15)	augmented sixth	dissonant atonal
si b	9/5	minor seventh	R. C.
si	13/7	major seventh	R. C.
si # = do b	(27/14)	augmented seventh	dissonant atonal

(In particular $(\sqrt[19]{2})^5 = 1.200\ldots =$ about 6/5).

The above intervals can be classified according to simplicity in:

HIGHLY CONSONANT: unison (1), fifth (3/2) and its inversion: fourth.

VERY CONSONANT: major sixth (5/3), major third (5/4), and their inversions: minor third, minor sixth.

RATHER CONSONANT: minor seventh (9/5), diminished fourth (9/7), major seventh (13/7), diminished fifth (13/9), and their inversions: second, augmented fifth, diminished second, augmented fourth.

DISSONANT: Chromatic, augmented second, augmented sixth, augmented seventh.

The classification in tonal-atonal will be defined in next section. (Other subdivisions of the octave, like in 31 tones, have been experimented).

3. - Two theorems discriminating dodecaphonic and enneadecaphonic scales.

From now on we understand to identify tones (of different octaves) with the same name.

In eather scale we follow the traditional

DEFINITION: *Major tonality of a key* = the tone called key and its second, major third, fourth, fifth, major sixth, major seventh. *Minor tonality of a key* = the tone called key and its second, minor third, fourth, fifth, minor sixth, minor seventh.

DEFINITION: An interval between two tones is called « *atonal* » if there exist no tonality to which both tones simultaneously belong.

THEOREM I: *In the dodecaphonic scale there is no « atonal » interval. In the enneadecaphonic scale the « atonal » intervals are: chromatic, augmented second, diminished fourth, augmented fifth, augmented sixth, augmented seventh.*

The proof follows from the fact that in the dodecaphonic scale between the seven tones of a tonality one finds samples of every (dodecaphonic) interval, which is *not* the case with enneadecaphonic scale and intervals.

Another traditional

DEFINITION: *Modulation* = circular permutation of the (12 or 19) names of tones, i.e. a shift on the key-board.

DEFINITION: A non empty subset of tones is called a *chord* if it contains no more than one dissonant couple and:

(1) No two tones of the chord have distance smaller than a « second ».

(2) No three tones of the chord lie within a « diminished fourth ».

(3) No four tones of the chord lie within a « fifth ».

DEFINITION: A chord is called « *monovalent* » if there exists no modulation (except the identity) which transforms the chord in itself.

THEOREM II: *In the dodecaphonic scale non « monovalent » chords are: the diminished seventh (invariant under modulations by minor thirds) and the augmented triad on major tonic (invariant under modulations by major thirds). In the enneadecaphonic scale every chord is « monovalent ».*

The last statement follows from the Lagrange theorem on groups: The modulations leaving invariant a chord would form a subgroup whose order should be a factor of the prime number 19.

4. - The enneadecaphonic scale.

A *restricted* version of the scale presented in sec. 2 sound preferable: The following *12 tones* are selected out of the 19

do re b re mi b mi fa fa # sol la b la si b si

(From a practical point of view this amounts to *just a different way of tuning* a standard 12-keys instrument).

With this « enneadecaphonic » scale the *allowed tonalities* are:

Major tonalities: sol, do, fa, si b, mi b, la b

Minor tonalities: mi, la, re, sol, do, fa

The properties of the « enneadecaphonic general » scale discussed in sec. 3 still hold for the « enneadecaphonic » scale here presented. (Indeed for every enneadecaphonic interval listed in sec. 2 one can still find in this scale a couple of tones which gives that interval).

5. May classic music have enneadecaphonic performance?

This problem has not yet been fully investigated.

A trivial limitation follows from the fact that only 6 major and 6 minor tonalities are now allowed (see sec. 4). Deeper problems are based on the new patterns presented by the consonant, quasi-consonant, and (actually) dissonant intervals (see sec. 2), and by the two theorems (sec. 3) about atonal intervals (which necessarily bring outside of the tonality) and monovalent chords (which do not allow modulations based on ambiguous interpretations of chords).

Thus, only some portions of classic music, only after suitable adjustments, may have enneadecaphonic performance. In fig. 1 few measures of the Beethoven's moonlight sonata are shown with required adjustments: the tonality has been changed and a note which in the enneadecaphonic scale would give rise to an unpleasant augmented second has been changed (see correction marked). Surprisingly, just two measures after this crucial point the music (just because of the powerful role given to minor thirds?) seems to be made in order to suggest performance in the enneadecaphonic scale.

6. - Mathematical rules of tonal composition.

DEFINITION: A *chord* is called « *tonal* » if its tones lie within a « tonality » (see sec. 3), otherwise it is called « *atonal* ».

In this section we use only « tonal » chords and we accept harmonic transitions based on the classification of the intervals given in sec. 2. An admissible sequence of tonal chords (each one with marked tonality minor — or major +) is shown in fig. 2a. Taking this kind of harmonic sequence as (something more than an) accompaniment, one can compose a related melody as follows:

DEFINITION: Among the 7 tones of the tonality of a chord, those which belong to the chord or which would *integrate* it into a still admissible chord will be called « *fundamental* », and the other tones (of the tonality) which are at the distance of a « fourth » or « fifth » from some tone of the chord will be called « *passing tones* », (as well as non dissonant atonal integrants).

Making use in each measure of the « fundamental » tones of the local chord, and following the transition-probability-matrix between melodic patterns shown in fig. 2b, one can associate to the previous harmony the melody in fig. 2c. If furthermore the notes marked by * in fig. 2b are allowed to be « passing tones » the composition in fig. 2d (which is an approximation of a well known theme of the seventh symphony of Beethoven) turns out to be compatible with the rules mentioned in this paper.

This version sounds preferable when the instruments are tuned according to the enneadecaphonic scale.

7. - An example of enneadecaphonic music.

As pointed out in sec. 3 a *new possibility* offered by *enneadecaphonic* music is the use of « *atonal* » intervals (which *avoid unwonted resolutions*ı. In this paper the use of *atonal* chords is shown by examples (see « Disgelo enneadecafonico »). This composition turns out to loose its musical meaning when played with a standard dodecaphonically tuned instrument.

BIBLIOGRAPHY

A paper about *enuneadecaphonic music* and related mathematical rules, written in collaboration with M. GHISLANDI, will soon appear.

Fig. 1

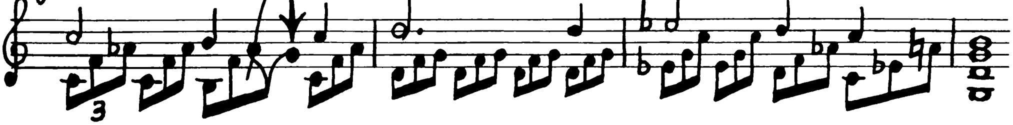

Fig. 2a

Fig. 2b

to:

from:	α	β	γ	δ	ε
α		1.			
β			1.		
γ		.5		.5	
δ	.25		.25		.5
ε				1.	

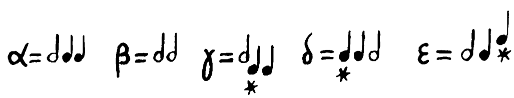

Fig. 2c

Fig. 2d


E. Gagliardo - "Disgelo enneadecafonico"
(Caution: Not for dodecaphonically tuned instruments)

E.G. "Danza sulla scalinata enneadecafonica"

E. G. "Sonata al chiaro di UFO"